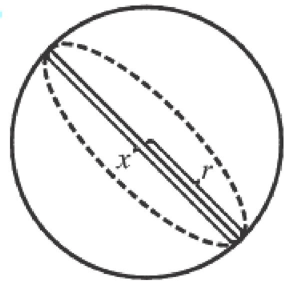
1. $V = x^3 \quad \Rightarrow \quad \dfrac{dV}{dt} = \dfrac{dV}{dx}\dfrac{dx}{dt} = 3x^2\,\dfrac{dx}{dt}$

4. $A = \ell w \quad \Rightarrow \quad \dfrac{dA}{dt} = \ell \cdot \dfrac{dw}{dt} + w \cdot \dfrac{d\ell}{dt} = 20(3) + 10(8) = 140\ \text{cm}^2/\text{s}.$

12. (a) Given: the rate of decrease of the surface area is 1 cm^2/min. If we let t be time (in minutes) and S be the surface area (in cm^2), then we are given that $dS/dt = -1\ \text{cm}^2/\text{s}$.

(b) Unknown: the rate of decrease of the diameter when the diameter is 10 cm. If we let x be the diameter, then we want to find dx/dt when $x = 10$ cm.

(c)



(d) If the radius is r and the diameter $x = 2r$, then $r = \frac{1}{2}x$ and

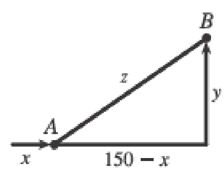
$$S = 4\pi r^2 = 4\pi\left(\tfrac{1}{2}x\right)^2 = \pi x^2 \quad \Rightarrow \quad \frac{dS}{dt} = \frac{dS}{dx}\frac{dx}{dt} = 2\pi x\,\frac{dx}{dt}.$$

(e) $-1 = \dfrac{dS}{dt} = 2\pi x\,\dfrac{dx}{dt} \quad \Rightarrow \quad \dfrac{dx}{dt} = -\dfrac{1}{2\pi x}$. When $x = 10$, $\dfrac{dx}{dt} = -\dfrac{1}{20\pi}$. So the rate of decrease is $\dfrac{1}{20\pi}$ cm/min.

14. (a) Given: at noon, ship A is 150 km west of ship B; ship A is sailing east at 35 km/h, and ship B is sailing north at 25 km/h. If we let t be time (in hours), x be the distance traveled by ship A (in km), and y be the distance traveled by ship B (in km), then we are given that $dx/dt = 35$ km/h and $dy/dt = 25$ km/h.

(b) Unknown: the rate at which the distance between the ships is changing at 4:00 PM. If we let z be the distance between the ships, then we want to find dz/dt when $t = 4$ h.

(c)



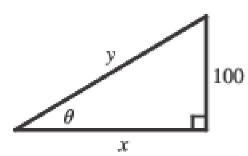
(d) $z^2 = (150 - x)^2 + y^2 \quad \Rightarrow \quad 2z\,\dfrac{dz}{dt} = 2(150 - x)\left(-\dfrac{dx}{dt}\right) + 2y\,\dfrac{dy}{dt}$

(e) At 4:00 PM, $x = 4(35) = 140$ and $y = 4(25) = 100 \quad \Rightarrow \quad z = \sqrt{(150 - 140)^2 + 100^2} = \sqrt{10{,}100}$.

So $\dfrac{dz}{dt} = \dfrac{1}{z}\left[(x - 150)\,\dfrac{dx}{dt} + y\,\dfrac{dy}{dt}\right] = \dfrac{-10(35) + 100(25)}{\sqrt{10{,}100}} = \dfrac{215}{\sqrt{101}} \approx 21.4$ km/h.

28. We are given $dx/dt = 8$ ft/s. $\cot\theta = \dfrac{x}{100} \quad \Rightarrow \quad x = 100\cot\theta \quad \Rightarrow$

$\dfrac{dx}{dt} = -100\csc^2\theta\,\dfrac{d\theta}{dt} \quad \Rightarrow \quad \dfrac{d\theta}{dt} = -\dfrac{\sin^2\theta}{100}\cdot 8$. When $y = 200$, $\sin\theta = \dfrac{100}{200} = \dfrac{1}{2} \quad \Rightarrow$



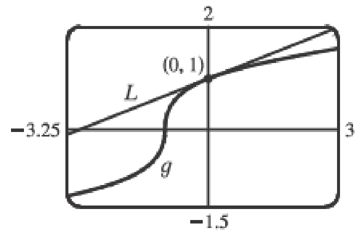
$\dfrac{d\theta}{dt} = -\dfrac{(1/2)^2}{100}\cdot 8 = -\dfrac{1}{50}$ rad/s. The angle is decreasing at a rate of $\frac{1}{50}$ rad/s.

2. $f(x) = \ln x \quad \Rightarrow \quad f'(x) = 1/x$, so $f(1) = 0$ and $f'(1) = 1$. Thus, $L(x) = f(1) + f'(1)\,(x - 1) = 0 + 1(x - 1) = x - 1$.

6. $g(x) = \sqrt[3]{1+x} = (1+x)^{1/3} \quad \Rightarrow \quad g'(x) = \frac{1}{3}(1+x)^{-2/3}$, so $g(0) = 1$ and $g'(0) = \frac{1}{3}$. Therefore, $\sqrt[3]{1+x} = g(x) \approx g(0) + g'(0)(x-0) = 1 + \frac{1}{3}x$.

So $\sqrt[3]{0.95} = \sqrt[3]{1+(-0.05)} \approx 1 + \frac{1}{3}(-0.05) = 0.98\overline{3}$,

and $\sqrt[3]{1.1} = \sqrt[3]{1+0.1} \approx 1 + \frac{1}{3}(0.1) = 1.0\overline{3}$.



30. If $y = x^6$, $y' = 6x^5$ and the tangent line approximation at $(1, 1)$ has slope 6. If the change in x is 0.01, the change in y on the tangent line is 0.06, and approximating $(1.01)^6$ with 1.06 is reasonable.

32. (a) $f(x) = (x-1)^2 \quad \Rightarrow \quad f'(x) = 2(x-1)$, so $f(0) = 1$ and $f'(0) = -2$.

Thus, $f(x) \approx L_f(x) = f(0) + f'(0)(x-0) = 1 - 2x$.

$g(x) = e^{-2x} \quad \Rightarrow \quad g'(x) = -2e^{-2x}$, so $g(0) = 1$ and $g'(0) = -2$.

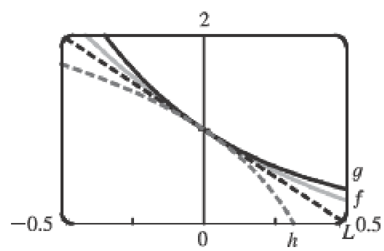
Thus, $g(x) \approx L_g(x) = g(0) + g'(0)(x-0) = 1 - 2x$.

$h(x) = 1 + \ln(1-2x) \quad \Rightarrow \quad h'(x) = \dfrac{-2}{1-2x}$, so $h(0) = 1$ and $h'(0) = -2$.

Thus, $h(x) \approx L_h(x) = h(0) + h'(0)(x-0) = 1 - 2x$.

Notice that $L_f = L_g = L_h$. This happens because f, g, and h have the same function values and the same derivative values at $a = 0$.

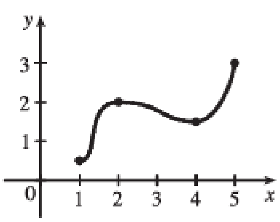
(b) The linear approximation appears to be the best for the function f since it is closer to f for a larger domain than it is to g and h. The approximation looks worst for h since h moves away from L faster than f and g do.



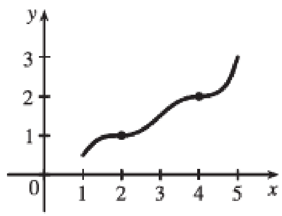
2. (a) The Extreme Value Theorem

(b) See the Closed Interval Method.

8. Absolute minimum at 1, absolute maximum at 5, local maximum at 2, local minimum at 4



10. f has no local maximum or minimum, but 2 and 4 are critical numbers



32. $f(x) = x^3 + x^2 + x \quad \Rightarrow \quad f'(x) = 3x^2 + 2x + 1.\quad f'(x) = 0 \quad \Rightarrow \quad 3x^2 + 2x + 1 = 0 \quad \Rightarrow \quad x = \dfrac{-2 \pm \sqrt{4 - 12}}{6}$.

Neither of these is a real number. Thus, there are no critical numbers.

34. $g(t) = |3t - 4| = \begin{cases} 3t - 4 & \text{if } 3t - 4 \geq 0 \\ -(3t - 4) & \text{if } 3t - 4 < 0 \end{cases} = \begin{cases} 3t - 4 & \text{if } t \geq \frac{4}{3} \\ 4 - 3t & \text{if } t < \frac{4}{3} \end{cases}$

$g'(t) = \begin{cases} 3 & \text{if } t > \frac{4}{3} \\ -3 & \text{if } t < \frac{4}{3} \end{cases}$ and $g'(t)$ does not exist at $t = \frac{4}{3}$, so $t = \frac{4}{3}$ is a critical number.

52. $f(x) = (x^2 - 1)^3$, $[-1, 2]$. $f'(x) = 3(x^2 - 1)^2(2x) = 6x(x + 1)^2(x - 1)^2 = 0 \quad \Leftrightarrow \quad x = -1, 0, 1$. $f(\pm 1) = 0$, $f(0) = -1$, and $f(2) = 27$. So $f(2) = 27$ is the absolute maximum value and $f(0) = -1$ is the absolute minimum value.