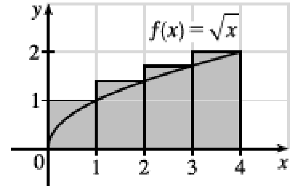
4. (a) $R_4 = \sum_{i=1}^{4} f(x_i)\,\Delta x \qquad \left[\Delta x = \frac{4-0}{4} = 1\right]$

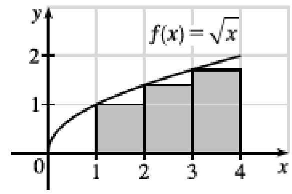
$$= f(x_1)\cdot 1 + f(x_2)\cdot 1 + f(x_3)\cdot 1 + f(x_4)\cdot 1$$
$$= f(1) + f(2) + f(3) + f(4)$$
$$= \sqrt{1} + \sqrt{2} + \sqrt{3} + \sqrt{4} \approx 6.1463$$



Since f is *increasing* on $[0, 4]$, R_4 is an *overestimate*.

(b) $L_4 = \sum_{i=1}^{4} f(x_{i-1})\,\Delta x = f(x_0)\cdot 1 + f(x_1)\cdot 1 + f(x_2)\cdot 1 + f(x_3)\cdot 1$

$$= f(0) + f(1) + f(2) + f(3)$$
$$= \sqrt{0} + \sqrt{1} + \sqrt{2} + \sqrt{3} \approx 4.1463$$



Since f is *increasing* on $[0, 4]$, L_4 is an *underestimate*.

17. $f(x) = \sqrt[4]{x}$, $1 \le x \le 16$. $\Delta x = (16-1)/n = 15/n$ and $x_i = 1 + i\,\Delta x = 1 + 15i/n$.

$$A = \lim_{n\to\infty} R_n = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x = \lim_{n\to\infty} \sum_{i=1}^{n} \sqrt[4]{1 + \frac{15i}{n}} \cdot \frac{15}{n}.$$

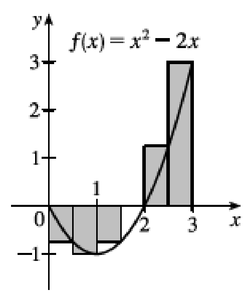
18. $f(x) = \frac{\ln x}{x}$, $3 \le x \le 10$. $\Delta x = (10-3)/n = 7/n$ and $x_i = 3 + i\,\Delta x = 3 + 7i/n$.

$$A = \lim_{n\to\infty} R_n = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{\ln(3 + 7i/n)}{3 + 7i/n} \cdot \frac{7}{n}.$$

2. $f(x) = x^2 - 2x$, $0 \le x \le 3$. $\Delta x = \frac{b-a}{n} = \frac{3-0}{6} = \frac{1}{2}$.

Since we are using right endpoints, $x_i^* = x_i$.

$$R_6 = \sum_{i=1}^{6} f(x_i)\,\Delta x$$
$$= (\Delta x)\left[f(x_1) + f(x_2) + f(x_3) + f(x_4) + f(x_5) + f(x_6)\right]$$
$$= \tfrac{1}{2}\left[f(\tfrac{1}{2}) + f(1) + f(\tfrac{3}{2}) + f(2) + f(\tfrac{5}{2}) + f(3)\right]$$
$$= \tfrac{1}{2}\left(-\tfrac{3}{4} - 1 - \tfrac{3}{4} + 0 + \tfrac{5}{4} + 3\right) = \tfrac{1}{2}\left(\tfrac{7}{4}\right) = \tfrac{7}{8}$$



The Riemann sum represents the sum of the areas of the two rectangles above the x-axis minus the sum of the areas of the three rectangles below the x-axis; that is, the *net area* of the rectangles with respect to the x-axis.

10. $\Delta x = (\pi/2 - 0)/4 = \frac{\pi}{8}$, so the endpoints are $0, \frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}$, and $\frac{\pi}{2}$, and the midpoints are $\frac{\pi}{16}, \frac{3\pi}{16}, \frac{5\pi}{16}$, and $\frac{7\pi}{16}$. The Midpoint Rule gives

$$\int_0^{\pi/2} \cos^4 x\,dx \approx \sum_{i=1}^{4} f(\overline{x}_i)\,\Delta x = \frac{\pi}{8}\left[\cos^4\left(\frac{\pi}{16}\right) + \cos^4\left(\frac{3\pi}{16}\right) + \cos^4\left(\frac{5\pi}{16}\right) + \cos^4\left(\frac{7\pi}{16}\right)\right] = \frac{\pi}{8}\left(\frac{3}{2}\right) \approx 0.5890.$$

18. On $[\pi, 2\pi]$, $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{\cos x_i}{x_i}\,\Delta x = \int_{\pi}^{2\pi} \frac{\cos x}{x}\,dx.$

22. $\int_1^4 (x^2+2x-5)\,dx = \lim\limits_{n\to\infty} \sum\limits_{i=1}^n f(x_i)\Delta x \qquad [\Delta x = 3/n \text{ and } x_i = 1+3i/n]$

$$= \lim_{n\to\infty} \sum_{i=1}^n \left[\left(1+\frac{3i}{n}\right)^2 + 2\left(1+\frac{3i}{n}\right) - 5\right]\left(\frac{3}{n}\right)$$

$$= \lim_{n\to\infty} \frac{3}{n}\left[\sum_{i=1}^n \left(1+\frac{6i}{n}+\frac{9i^2}{n^2}+2+\frac{6i}{n}-5\right)\right]$$

$$= \lim_{n\to\infty} \frac{3}{n}\left[\sum_{i=1}^n \left(\frac{9}{n^2}\cdot i^2 + \frac{12}{n}\cdot i - 2\right)\right] = \lim_{n\to\infty} \frac{3}{n}\left[\frac{9}{n^2}\sum_{i=1}^n i^2 + \frac{12}{n}\sum_{i=1}^n i - \sum_{i=1}^n 2\right]$$

$$= \lim_{n\to\infty} \left(\frac{27}{n^3}\cdot\frac{n(n+1)(2n+1)}{6} + \frac{36}{n^2}\cdot\frac{n(n+1)}{2} - \frac{6}{n}\cdot n\right)$$

$$= \lim_{n\to\infty} \left(\frac{9}{2}\cdot\frac{n+1}{n}\cdot\frac{2n+1}{n} + 18\cdot\frac{n+1}{n} - 6\right)$$

$$= \lim_{n\to\infty} \left[\frac{9}{2}\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right) + 18\left(1+\frac{1}{n}\right) - 6\right] = \tfrac{9}{2}\cdot 1\cdot 2 + 18\cdot 1 - 6 = 21$$

34. (a) $\int_0^2 g(x)\,dx = \frac{1}{2}\cdot 4\cdot 2 = 4$ [area of a triangle]

(b) $\int_2^6 g(x)\,dx = -\frac{1}{2}\pi(2)^2 = -2\pi$ [negative of the area of a semicircle]

(c) $\int_6^7 g(x)\,dx = \frac{1}{2}\cdot 1\cdot 1 = \frac{1}{2}$ [area of a triangle]

$\int_0^7 g(x)\,dx = \int_0^2 g(x)\,dx + \int_2^6 g(x)\,dx + \int_6^7 g(x)\,dx = 4 - 2\pi + \frac{1}{2} = 4.5 - 2\pi$

40. $\int_0^{10} |x-5|\,dx$ can be interpreted as the sum of the areas of the two shaded triangles; that is, $2\left(\frac{1}{2}\right)(5)(5) = 25$.