

14. (a) Let  $H(t) = 10t - 1.86t^2$ .

$$\begin{aligned} v(1) &= \lim_{h \rightarrow 0} \frac{H(1+h) - H(1)}{h} = \lim_{h \rightarrow 0} \frac{[10(1+h) - 1.86(1+h)^2] - (10 - 1.86)}{h} \\ &= \lim_{h \rightarrow 0} \frac{10 + 10h - 1.86(1 + 2h + h^2) - 10 + 1.86}{h} \\ &= \lim_{h \rightarrow 0} \frac{10 + 10h - 1.86 - 3.72h - 1.86h^2 - 10 + 1.86}{h} \\ &= \lim_{h \rightarrow 0} \frac{6.28h - 1.86h^2}{h} = \lim_{h \rightarrow 0} (6.28 - 1.86h) = 6.28 \end{aligned}$$

The velocity of the rock after one second is 6.28 m/s.

$$\begin{aligned} \text{(b) } v(a) &= \lim_{h \rightarrow 0} \frac{H(a+h) - H(a)}{h} = \lim_{h \rightarrow 0} \frac{[10(a+h) - 1.86(a+h)^2] - (10a - 1.86a^2)}{h} \\ &= \lim_{h \rightarrow 0} \frac{10a + 10h - 1.86(a^2 + 2ah + h^2) - 10a + 1.86a^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{10a + 10h - 1.86a^2 - 3.72ah - 1.86h^2 - 10a + 1.86a^2}{h} = \lim_{h \rightarrow 0} \frac{10h - 3.72ah - 1.86h^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{h(10 - 3.72a - 1.86h)}{h} = \lim_{h \rightarrow 0} (10 - 3.72a - 1.86h) = 10 - 3.72a \end{aligned}$$

The velocity of the rock when  $t = a$  is  $(10 - 3.72a)$  m/s.

- (c) The rock will hit the surface when  $H = 0 \Leftrightarrow 10t - 1.86t^2 = 0 \Leftrightarrow t(10 - 1.86t) = 0 \Leftrightarrow t = 0$  or  $1.86t = 10$ .

The rock hits the surface when  $t = 10/1.86 \approx 5.4$  s.

- (d) The velocity of the rock when it hits the surface is  $v(\frac{10}{1.86}) = 10 - 3.72(\frac{10}{1.86}) = 10 - 20 = -10$  m/s.

18. (a) Since  $g(5) = -3$ , the point  $(5, -3)$  is on the graph of  $g$ . Since  $g'(5) = 4$ , the slope of the tangent line at  $x = 5$  is 4.

Using the point-slope form of a line gives us  $y - (-3) = 4(x - 5)$ , or  $y = 4x - 23$ .

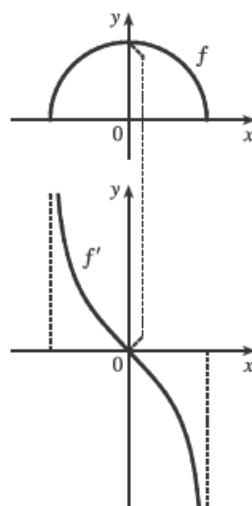
- (b) Since  $(4, 3)$  is on  $y = f(x)$ ,  $f(4) = 3$ . The slope of the tangent line between  $(0, 2)$  and  $(4, 3)$  is  $\frac{1}{4}$ , so  $f'(4) = \frac{1}{4}$ .

$$\begin{aligned} 28. f'(a) &= \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} = \lim_{h \rightarrow 0} \frac{\frac{(a+h)^2 + 1}{(a+h) - 2} - \frac{a^2 + 1}{a - 2}}{h} \\ &= \lim_{h \rightarrow 0} \frac{(a^2 + 2ah + h^2 + 1)(a - 2) - (a^2 + 1)(a + h - 2)}{h(a + h - 2)(a - 2)} \\ &= \lim_{h \rightarrow 0} \frac{(a^3 - 2a^2 + 2a^2h - 4ah + ah^2 - 2h^2 + a - 2) - (a^3 + a^2h - 2a^2 + a + h - 2)}{h(a + h - 2)(a - 2)} \\ &= \lim_{h \rightarrow 0} \frac{a^2h - 4ah + ah^2 - 2h^2 - h}{h(a + h - 2)(a - 2)} = \lim_{h \rightarrow 0} \frac{h(a^2 - 4a + ah - 2h - 1)}{h(a + h - 2)(a - 2)} = \lim_{h \rightarrow 0} \frac{a^2 - 4a + ah - 2h - 1}{(a + h - 2)(a - 2)} \\ &= \frac{a^2 - 4a - 1}{(a - 2)^2} \end{aligned}$$

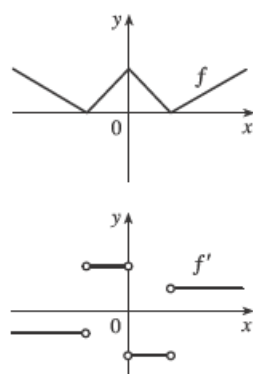
36. By Equation 3,  $\lim_{t \rightarrow 1} \frac{t^4 + t - 2}{t - 1} = f'(1)$ , where  $f(t) = t^4 + t$  and  $a = 1$ .

3. (a)' = II, since from left to right, the slopes of the tangents to graph (a) start out negative, become 0, then positive, then 0, then negative again. The actual function values in graph II follow the same pattern.
- (b)' = IV, since from left to right, the slopes of the tangents to graph (b) start out at a fixed positive quantity, then suddenly become negative, then positive again. The discontinuities in graph IV indicate sudden changes in the slopes of the tangents.
- (c)' = I, since the slopes of the tangents to graph (c) are negative for  $x < 0$  and positive for  $x > 0$ , as are the function values of graph I.
- (d)' = III, since from left to right, the slopes of the tangents to graph (d) are positive, then 0, then negative, then 0, then positive, then 0, then negative again, and the function values in graph III follow the same pattern.

7.



9.



$$\begin{aligned}
 24. f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{(x+h+\sqrt{x+h}) - (x+\sqrt{x})}{h} \\
 &= \lim_{h \rightarrow 0} \left( \frac{h}{h} + \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}} \right) = \lim_{h \rightarrow 0} \left[ 1 + \frac{(x+h) - x}{h(\sqrt{x+h} + \sqrt{x})} \right] \\
 &= \lim_{h \rightarrow 0} \left( 1 + \frac{1}{\sqrt{x+h} + \sqrt{x}} \right) = 1 + \frac{1}{\sqrt{x} + \sqrt{x}} = 1 + \frac{1}{2\sqrt{x}}
 \end{aligned}$$

Domain of  $f = [0, \infty)$ , domain of  $f' = (0, \infty)$ .

$$8. f(t) = \frac{1}{2}t^6 - 3t^4 + t \Rightarrow f'(t) = \frac{1}{2}(6t^5) - 3(4t^3) + 1 = 3t^5 - 12t^3 + 1$$

$$14. R(t) = 5t^{-3/5} \Rightarrow R'(t) = 5 \left[ -\frac{3}{5}t^{(-3/5)-1} \right] = -3t^{-8/5}$$

$$32. y = e^{x+1} + 1 = e^x e^1 + 1 = e \cdot e^x + 1 \Rightarrow y' = e \cdot e^x = e^{x+1}$$

$$36. y = (1+2x)^2 = 1 + 4x + 4x^2 \Rightarrow y' = 4 + 8x. \text{ At } (1, 9), y' = 12 \text{ and an equation of the tangent line is}$$

$y - 9 = 12(x - 1)$  or  $y = 12x - 3$ . The slope of the normal line is  $-\frac{1}{12}$  (the negative reciprocal of 12) and an equation of the normal line is  $y - 9 = -\frac{1}{12}(x - 1)$  or  $y = -\frac{1}{12}x + \frac{109}{12}$ .

$$50. (a) s = 2t^3 - 7t^2 + 4t + 1 \Rightarrow v(t) = s'(t) = 6t^2 - 14t + 4 \Rightarrow a(t) = v'(t) = 12t - 14$$

$$(b) a(1) = 12 - 14 = -2 \text{ m/s}^2$$

(c)

