4. $\lim\limits_{x\to 2} \dfrac{2x^2+1}{x^2+6x-4} = \dfrac{\lim\limits_{x\to 2}(2x^2+1)}{\lim\limits_{x\to 2}(x^2+6x-4)}$ [Limit Law 5]

$= \dfrac{2\lim\limits_{x\to 2} x^2 + \lim\limits_{x\to 2} 1}{\lim\limits_{x\to 2} x^2 + 6\lim\limits_{x\to 2} x - \lim\limits_{x\to 2} 4}$ [2, 1, and 3]

$= \dfrac{2(2)^2+1}{(2)^2+6(2)-4} = \dfrac{9}{12} = \dfrac{3}{4}$ [9, 7, and 8]

6. $\lim\limits_{t\to -1}(t^2+1)^3(t+3)^5 = \lim\limits_{t\to -1}(t^2+1)^3 \cdot \lim\limits_{t\to -1}(t+3)^5$ [Limit Law 4]

$= \left[\lim\limits_{t\to -1}(t^2+1)\right]^3 \cdot \left[\lim\limits_{t\to -1}(t+3)\right]^5$ [6]

$= \left[\lim\limits_{t\to -1} t^2 + \lim\limits_{t\to -1} 1\right]^3 \cdot \left[\lim\limits_{t\to -1} t + \lim\limits_{t\to -1} 3\right]^5$ [1]

$= \left[(-1)^2+1\right]^3 \cdot [-1+3]^5 = 8\cdot 32 = 256$ [9, 7, and 8]

8. $\lim\limits_{u\to -2}\sqrt{u^4+3u+6} = \sqrt{\lim\limits_{u\to -2}(u^4+3u+6)}$ [11]

$= \sqrt{\lim\limits_{u\to -2} u^4 + 3\lim\limits_{u\to -2} u + \lim\limits_{u\to -2} 6}$ [1, 2, and 3]

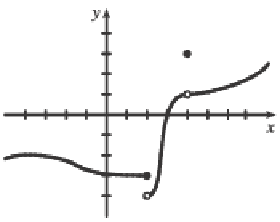
$= \sqrt{(-2)^4+3(-2)+6}$ [9, 8, and 7]

$= \sqrt{16-6+6} = \sqrt{16} = 4$

42. Since $|x| = -x$ for $x<0$, we have $\lim\limits_{x\to -2}\dfrac{2-|x|}{2+x} = \lim\limits_{x\to -2}\dfrac{2-(-x)}{2+x} = \lim\limits_{x\to -2}\dfrac{2+x}{2+x} = \lim\limits_{x\to -2} 1 = 1.$

4. g is continuous on $[-4,-2)$, $(-2,2)$, $[2,4)$, $(4,6)$, and $(6,8)$.

6.



10. $\lim\limits_{x\to 4} f(x) = \lim\limits_{x\to 4}\left(x^2+\sqrt{7-x}\right) = \lim\limits_{x\to 4} x^2 + \sqrt{\lim\limits_{x\to 4} 7 - \lim\limits_{x\to 4} x} = 4^2 + \sqrt{7-4} = 16+\sqrt{3} = f(4).$

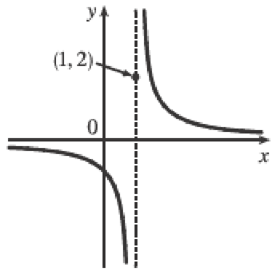
By the definition of continuity, f is continuous at $a=4$.

14. For $a<3$, we have $\lim\limits_{x\to a} g(x) = \lim\limits_{x\to a} 2\sqrt{3-x} = 2\lim\limits_{x\to a}\sqrt{3-x}$ [Limit Law 3] $= 2\sqrt{\lim\limits_{x\to a}(3-x)}$ [11]

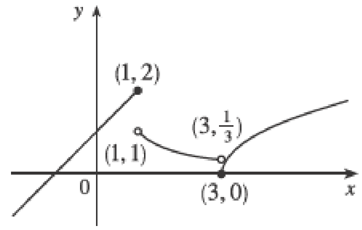
$= 2\sqrt{\lim\limits_{x\to a} 3 - \lim\limits_{x\to a} x}$ [2] $= 2\sqrt{3-a}$ [7 and 8] $= g(a)$, so g is continuous at $x=a$ for every a in $(-\infty, 3)$.

Also, $\lim\limits_{x\to 3^-} g(x) = 0 = g(3)$, so g is continuous from the left at 3. Thus, g is continuous on $(-\infty, 3]$.

16. $f(x) = \begin{cases} 1/(x-1) & \text{if } x \neq 1 \\ 2 & \text{if } x = 1 \end{cases}$ is discontinuous at 1 because $\lim_{x \to 1} f(x)$ does not exist.



38. $f(x) = \begin{cases} x+1 & \text{if } x \leq 1 \\ 1/x & \text{if } 1 < x < 3 \\ \sqrt{x-3} & \text{if } x \geq 3 \end{cases}$



f is continuous on $(-\infty, 1)$, $(1, 3)$, and $(3, \infty)$, where it is a polynomial, a rational function, and a composite of a root function with a polynomial, respectively. Now $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (x+1) = 2$ and $\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (1/x) = 1$, so f is discontinuous at 1. Since $f(1) = 2$, f is continuous from the left at 1. Also, $\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} (1/x) = 1/3$, and $\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} \sqrt{x-3} = 0 = f(3)$, so f is discontinuous at 3, but it is continuous from the right at 3.

48. $f(x) = \sqrt[3]{x} + x - 1$ is continuous on the interval $[0, 1]$, $f(0) = -1$, and $f(1) = 1$. Since $-1 < 0 < 1$, there is a number c in $(0, 1)$ such that $f(c) = 0$ by the Intermediate Value Theorem. Thus, there is a root of the equation $\sqrt[3]{x} + x - 1 = 0$, or $\sqrt[3]{x} = 1 - x$, in the interval $(0, 1)$.

4. (a) (i) Using Definition 1 with $f(x) = x - x^3$ and $P(1, 0)$,

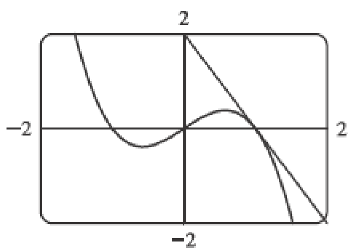
$$m = \lim_{x \to 1} \frac{f(x) - 0}{x - 1} = \lim_{x \to 1} \frac{x - x^3}{x - 1} = \lim_{x \to 1} \frac{x(1 - x^2)}{x - 1} = \lim_{x \to 1} \frac{x(1 + x)(1 - x)}{x - 1}$$
$$= \lim_{x \to 1} [-x(1 + x)] = -1(2) = -2$$

(ii) Using Equation 2 with $f(x) = x - x^3$ and $P(1, 0)$,

$$m = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h} = \lim_{h \to 0} \frac{f(1 + h) - f(1)}{h} = \lim_{h \to 0} \frac{[(1 + h) - (1 + h)^3] - 0}{h}$$
$$= \lim_{h \to 0} \frac{1 + h - (1 + 3h + 3h^2 + h^3)}{h} = \lim_{h \to 0} \frac{-h^3 - 3h^2 - 2h}{h} = \lim_{h \to 0} \frac{h(-h^2 - 3h - 2)}{h}$$
$$= \lim_{h \to 0} (-h^2 - 3h - 2) = -2$$

(b) An equation of the tangent line is $y - f(a) = f'(a)(x - a) \quad \Rightarrow \quad y - f(1) = f'(1)(x - 1) \quad \Rightarrow \quad y - 0 = -2(x - 1)$, or $y = -2x + 2$.

(c)



The graph of $y = -2x + 2$ is tangent to the graph of $y = x - x^3$ at the point $(1, 0)$. Now zoom in toward the point $(1, 0)$ until the cubic and the tangent line are indistinguishable.

6. Using (1),

$$m = \lim_{x \to -1} \frac{(2x^3 - 5x) - 3}{x - (-1)} = \lim_{x \to -1} \frac{2x^3 - 5x - 3}{x + 1} = \lim_{x \to -1} \frac{(2x^2 - 2x - 3)(x + 1)}{x + 1} = \lim_{x \to -1} (2x^2 - 2x - 3) = 1.$$

Tangent line: $y - 3 = 1\,[x - (-1)] \quad \Leftrightarrow \quad y = x + 4$