Problem 143 (Exercise 13, page 314)**.** Use these guidelines to sketch the curve

$$f(x) = \frac{x}{x^2 + 9}.$$

Solution.

[lots of workings]

Problem 144 (Exercise 26, page 314)**.** Use these guidelines to sketch the curve

$$y = \frac{x}{\sqrt{x^2 - 1}}.$$

Solution.

[lots of workings]

4.7 Optimization Problems

Similar to related rates, these are the word problems corresponding to questions about maximum and minimum values of "real life" functions.

Steps in Solving Optimization Problems See text, page 322.

Problem 145 (Exercise 10, page 328)**.** A box with an open top is to be constructed from a square piece of cardboard, 3ft wide, by cutting out a square from each of the four corners and bending up the sides. Find the largest volume that such a box can have.

Solution.

[lots of workings]

Problem 146 (Exercise 5, page 328)**.** Find the dimensions of a rectangle with perimeter 100m whose area is as large as possible.

Solution.

[lots of workings]

Example 147 (Exercise 17, page 328)**.** Find the point on the line $y = 4x + 7$ that is closest to the origin (the point $(0, 0)$).

Solution.

4.9 Antiderivatives

In this section, we go backwards.

Definition 148. A function F is called an *antiderivative* of f on an interval I if $F'(x) = f(x)$ for all x in I.

Example 149. Consider the function

$$f(x) = 3x^2.$$

From experience, we recognize $f(x)$ as the derivative of x^3. However, the function

$$F(x) = x^3 + c$$

for any c is also an antiderivative of $f(x)$, as the derivative of any constant is 0.

Theorem 150. *If F is an antiderivative of f on an interval I, then the most general antiderivative of f on I is*

$$F(x) + c,$$

where c is an arbitrary constant.

Remark. This is true even if the function F is some other function plus a constant, as that constant plus an arbitrary constant acts just like an arbitrary constant.

Problem 151. Find the most general antiderivative of

$$f(x) = \frac{1}{2}x^2 - 2x + 6.$$

Solution.

Problem 152. Find the most general antiderivative of

$$f(x) = \cos(x).$$

Solution.

Problem 153. Find the most general antiderivative of

$$f(x) = x^n (n \neq 1).$$

Solution.

function	particular antiderivative
$cf(x)$	$cF(x)$
$f(x)+g(x)$	$F(x)+G(x)$
$x^n (n \neq 1)$	$\frac{x^n+1}{n+1}$
$\frac{1}{x}$	$\ln \lvert x \rvert$
e^x	e^x
$\cos(x)$	$\sin(x)$
$\sin(x)$	$-\cos(x)$
$\sec^2(x)$	$\tan(x)$
$\sec(x)\tan(x)$	$\sec(x)$
$\frac{1}{\sqrt{1-x^2}}$	$\arcsin(x)$
$\frac{1}{1+x^2}$	$\arctan(x)$

Figure 4.1: Some particular antiderivatives (if $F' = f$ and $G' = g$)

Problem 154. Find f if

$$f' = \sqrt{x}(6+5x), \;\; f(1) = 10.$$

Solution.

Antidifferentiation is particularly useful in analyzing motion of an object moving in a straight line. Recall if the object has position function $s(t)$, then the velocity function is $v(t) = s'(t)$. This means that the position function is the antiderivative of the velocity function. Also, the acceleration function is $a(t) = v'(t)$, which means that the velocity function is the antiderivative of the acceleration function.

Problem 155 (Exercise 60, page 346)**.** A particle is moving with

$$a(t) = \cos(t) + \sin(t),\ s(0) = 0,\ v(0) = 5.$$

Find the position of the particle.

Solution.