1. A coupon at a restaurant entitles me to a free appetizer, soup, or salad. The restaurant offers 8 appetizers, 5 salads, and 3 soups. How many different options do I have for my free item? [1]

$$8 + 5 + 3 = 16$$

2. Three officers—a president, a treasurer, and a secretary—are to be chosen from among four people: Ann, Bob, Cyd, and Dan. Supposed that, for various reasons, Ann cannot be president and either Cyd or Dan must be secretary. How many ways can the officers be chosen? [2]

   There are two solutions to this problem in the text, both with nice pictures and full explainations.

3. Three six-sided dice are rolled. Give an expression for the probability that they sum to 15? [3]

   The three dice have to show 663, 654 or 555 in some order to add to 15. There are thus $6 + 3 + 1 = 10$ ways, counting order, to roll the dice so that they sum to 15. As there are $6^3 = 216$ possible dice rolls, the probability of such a roll is

$$\frac{10}{216} = \frac{5}{108}.$$

4. Determine how many numbers from 0 to 9999, inclusive, are divisible by 2 or 5. [3]

   A number is divisible by 2 or 5 if and only if its last digit is in $\{0, 2, 4, 5, 6, 8\}$. There are thus $10 \times 10 \times 10 \times |\{0, 2, 4, 5, 6, 8\}| = 6000$ such numbers by the multiplication rule.

5. What is the probability of being dealt a three-of-a-kind in a 5 card poker hand? (Give an expression for your answer; do not work it out completely.) [3]

   A three-of-a-kind has three cards of the same denomination, and then two more cards, each of a different denomination. The number of three-of-a-kind hands is

$$\binom{13}{1}\binom{4}{3}\binom{12}{2}\binom{4}{1}^2.$$

   Therefore, the probability of being dealt one such hand out of the $\binom{52}{5}$ possible hands is

$$\frac{\binom{13}{1}\binom{4}{3}\binom{12}{2}\binom{4}{1}^2}{\binom{52}{5}}.$$

6. There are 8 types of pastries being served at a reception. How many ways can you eat 5 pieces, if we only care about what type is eaten? [2]

   The answer is $\binom{5+8-1}{5} = 792$.

7. How many (indistinguishable) words can be made using all the letters in DISCRETE? [2]

   There are 8 letters, of which two are the same. The answer is $\frac{8!}{2!}$.

8. Simplify [3]

$$\sum_{i=2}^{13} \binom{i}{2} = \binom{14}{3} = 364.$$

9. Simplify [2]

$$\sum_{i=0}^{13} \binom{13}{i} = 2^{13} = 8192$$

10. A fair coin is tossed 7 times. What is the expected number of heads? [2]

    There are numerous ways long and short to solve this problem. We argue that the expected number of heads equals the expected number of tails, and that the expected number of outcomes is 7, therefore $EV(H) = \frac{7}{2}$.

11. Let X and Y be events in the same sample space. Suppose $P(X|Y) = \frac{1}{5}$. [2]

    (a) If $P(X \cap Y) = \frac{1}{20}$, then what is $P(Y)$?

    (b) If X and Y are independent, then what is $P(X)$?

    For (a), $P(X \cap Y) = P(X|Y) \times P(Y)$, so

$$P(Y) = \frac{\frac{1}{20}}{\frac{1}{5}} = \frac{1}{4}.$$

    For (b), if X and Y are independent, then $P(X) = P(X|Y) = \frac{1}{5}$.

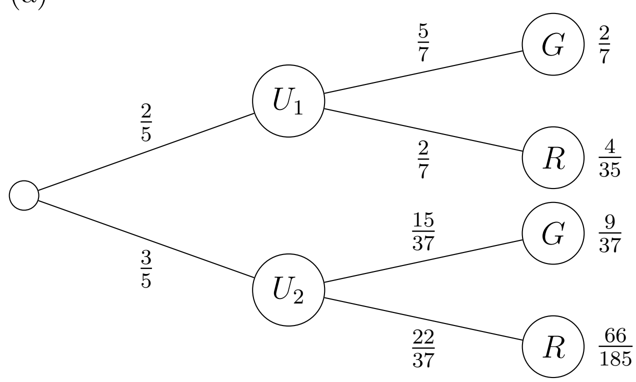
12. One urn contains 10 red balls and 25 green balls, and a second urn contains 22 red balls and 15 green balls. A ball is chosen as follows: First an urn is selected by tossing a loaded coin with probability 0.4 of landing heads up and probability 0.6 of landing tails up. If the coin lands heads up, the first urn is chosen; otherwise, the second urn is chosen. Then a ball is picked at random from the chosen urn. [5]

    (a) Draw a probability tree for the experiment.

    (b) What is the probability that the chosen ball is green?

    (c) If the chosen ball is green, what is the probability that it was picked from the first urn?

(a)



(b)

$$P(G) = \frac{2}{7} + \frac{9}{37}.$$

(c)

$$P(U_1|G) = \frac{\frac{2}{7}}{\frac{2}{7} + \frac{9}{37}} = \frac{74}{137}.$$

Total: 30