Due by 1559 AST Friday, April 1, 2011 — **Show your work**

CSCI/MATH 2113 Final Exam: Tuesday 12 April 2011 — 14:00 — Dalplex

1. Let $g(x) = (1 + x^2 + x^4)(1 + x + x^2)^2$ be the generating function for a_n. Find a_n for $n \geq 0$.

2. Build a generating function for the number of positive integer solutions to $n_1 + n_2 + n_3 + n_4 = r$, where n_1 is odd, n_2 is even, and n_4 is at most 4.

3. Build a generating function for a_r, the number of ways to make r cents change using pennies, nickels, dimes and quarters.

4. Find a generating function for a_r, the number of ways two six-sided dice can sum to r.

5. Let $g(x)$ be a generating function. Describe using a sentence, what the coefficient of x^n is in $\frac{g(x)}{1-x}$.

6. Prove or disprove:
$$\sum_{0 \leq k \leq n} \binom{n}{k}^2 = \binom{2n}{n}$$
(Hint: $\left(\frac{1}{1-x}\right)^2 = \frac{1}{(1-x)^2}$)

7. Five distinct bags each have 8 identical marbles in them. Using generating functions, find how many ways there are to select 20 marbles, if you take at least two from each bag?

8. Suppose $a_{n+1} = ba_n + c$ and $a_0 = d$. Then, using generating functions, find a general formula for a_n in terms of n (and b, c and d).

9. Let $a_n = a_{n-1} + n^3$ and $a_0 = 0$. Using generating functions, find a formula for a_n.