Due by 1559 AST Friday, January 28, 2011 — Show your work

1. A pizza restaurant offers 15 different toppings. How many different 4 topping pizzas are possible? (Note that you can order the same topping multiple times.)

2. A student goes to the local coffee shop to pick up a 20-pack of donut holes for breakfast. The shop offers 8 varieties of donut holes, but they only have 3 coconut donut holes left. How many different ways can the 20-pack be filled?

3. How many numbers from 100 to 999 (inclusive) have a hundreds digit that is at least the tens digit, and a tens digit that is at least the ones digit? (Note that both 100 and 999 satisfy this condition.)

4. How many integer solutions are there to $x_1 + x_2 + x_3 + x_4 = 21$ if $x_i \geq i$?

5. How many non-negative integer solutions are there to $x_1 + x_2 + x_3 + x_4 = 21$ if $6 \leq x_1 \leq 8$?

6. How many numbers from 1 to 999 have digits that sum to 18? (Hint: A number is divisible by 9 if and only if its digits add up to a multiple of 9.)

7. Prove Pascal's Theorem using a committee selection argument.

8. Compute the row of Pascal's triangle that corresponds to $n = 12$.

9. Prove that the number of ways to select an odd number of objects from a set of n objects is the same as the number of ways to select an even number of objects from a set of n objects.

10. Find $S_{4,i}$ for $0 \leq i \leq 4$.

11. Compute the probability of rolling two six-sided dice that sum to n for $2 \leq n \leq 12$. What is the probability that the sum of a dice roll is even? Justify your answer in two ways.

12. In a certain gambling game, the player rolls a pair of six-sided dice. If the player doesn't roll doubles he pays the sum of the dice to the dealer. If the player rolls doubles, then the dealer pays the player five times the sum of the dice. What is the expected payoff to the player?

13. For a particular disease, it has been determined that 2% of the population have the disease. There is a test for the disease that is 80% accurate (whether or not the person being tested has the disease). If a random person is tested for the disease and the test is positive, then how likely it is that the person tested actually has the disease?

14. There are two bags with marbles. Bag 1 contains 3 blue marbles and 5 red marbles. Bag 2 contains 5 yellow marbles and 2 red marbles. A marble is taken from Bag 1 and put in Bag 2 and then a marble is drawn from Bag 2. What is the probability that the marble drawn from Bag 2 is red?

15. An executive consisting of a president, vice-president, and treasurer is randomly chosen from a group of 5 men and 5 women. Given that at least one woman is on the executive, what is the probability that a woman is the president?