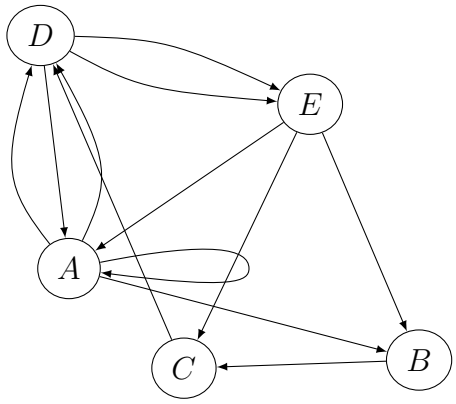
Due by 1230 AST Friday, February 18, 2011 — Show your work

1. Give an adjacency matrix for the following (directed) graph.



2. Draw a (directed) graph with the given adjacency matrix.

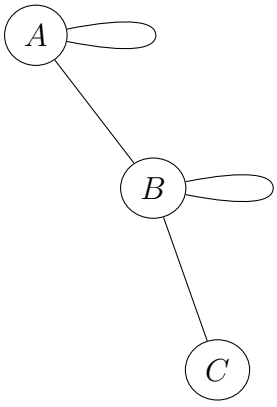
$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 2 & 0 \\ 0 & 2 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

3. Give adjacency matrices for

   (a) K_5

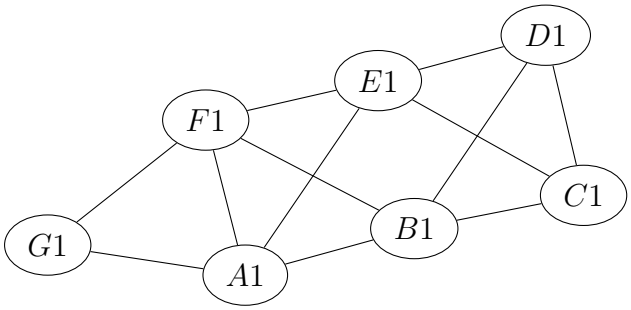
   (b) $K_{3,3}$

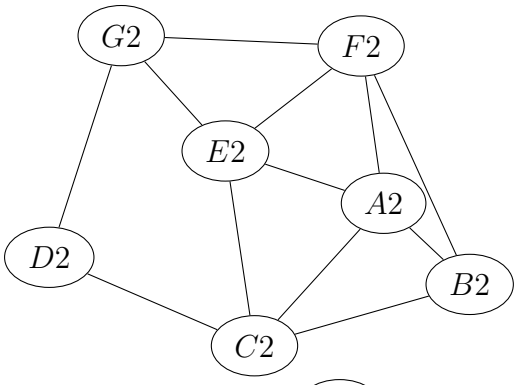
4. How many walks are there in the given graph from A to B of length 4?

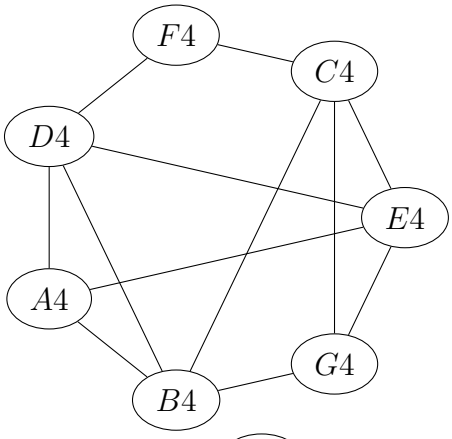


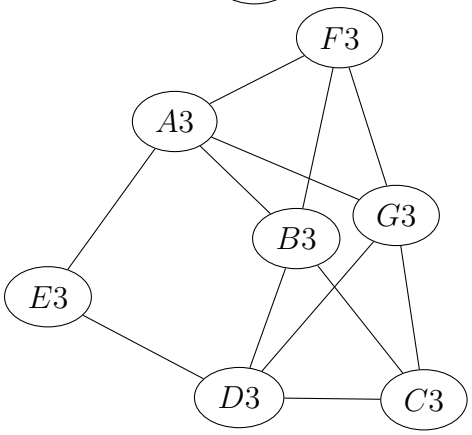
5. Prove the following: If A and B are symmetric matrices of size 2, then AB is also symmetric.

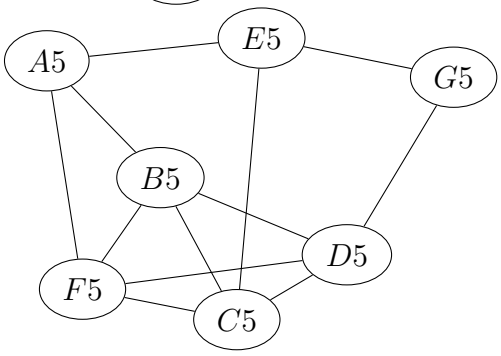
6. Of the following 5 graphs, which pairs, if any, are isomorphic? Give functions to show that two are isomorphic, or an argument that says that two are not isomorphic.











7. Draw all nonisomorphic graphs on 5 vertices with exactly 4 edges that are simple and disconnected.

8. Prove that "having a simple circuit of length 3" is an invariant for graph isomorphism.

9. What is the total degree of a tree with n vertices?

10. Describe the number of leaves in a rooted tree with respect to the number of non-parents.

11. Consider the following rooted tree.

    (a) How many terminal vertices are there?

    (b) How many parents are there?

    (c) What is the height of the tree?

    (d) How many children does the root have?

    (e) How many grandchildren does the root have?