Due by 1559 AST Friday, March 11, 2011 — **Show your work**

1. Algorithm A runs in n^2 steps for input of size n. Algorithm B runs in $1000 \log_2 n$ steps for input of size n. How big does the input have to be before we prefer B to A?

   For the following three questions, find an array (of distinct positive integers) of minimum size for which the given algorithm will run and make at least 20 comparisons, and give the number of comparisons.

2. Selection sort

3. Insertion sort

4. Merge sort

5. Insertion sort runs in $n^2 + n - 2$ steps for input of size n, according to the text. Also from the text, for input of size n, Merge sort takes between $\frac{1}{2} n \log_2 n$ and $2n \log_2 n$ steps. Assuming the slowest run time in the given range for merge sort, at what size for n do we prefer Merge sort?

6. For the following numbers (given in base 10) how many digits to the left of the decimal point are there in the (a) base 10 representation, (b) base 2 representation and (c) base e representation?

   (a) a trillion trillion (10^{18})

   (b) the 8th Mersenne prime ($2^{31} - 1$)

   (c) the largest known Fermat prime ($2^{2^4} + 1$)

7. How many multiplications are needed to compute x^n, if $n = 200+$ "the day of the month on which you were born"?