**Solutions to the problems from Assignment 2:**

6(a) There are two ways to do this. A proof by contradiction goes as follows:

Suppose that $\sqrt{6}-\sqrt{2}\leq 1$. Note that $6>2$, so $\sqrt{6}>\sqrt{2}$, so $\sqrt{6}-\sqrt{2}>0$, so if $\sqrt{6}-\sqrt{2}\leq 1$, then $(\sqrt{6}-\sqrt{2})^2\leq 1^2=1$. We derive that $6-2\sqrt{12}+2\leq 1$, so $8-1\leq 2\sqrt{12}$, so $7\leq 2\sqrt{12}$, so $49\leq 48$, which is clearly a contradiction, so $\sqrt{6}-\sqrt{2}>1$.

A direct proof goes as follows:

$48<49$, so $\sqrt{48}<\sqrt{49}$, so $2\sqrt{12}<7$, so $2\sqrt{12}<6+2-1$, so $1<6-2\sqrt{12}+2=(\sqrt{6}-\sqrt{2})^2$, so $1<\sqrt{6}-\sqrt{2}$.

7(b) In logical notation, this statement is

$$\forall n,k\in\mathbb{N}, n^k-n \text{ is a multiple of } k$$

or

$$\forall n,k\in\mathbb{N},\exists m\in\mathbb{N}, n^k-n=km$$

The negation of this statement is

$$\exists n,k\in\mathbb{N}, n^k-n \text{ is not a multiple of } k.$$

So in order to disprove this statement we need to find an example of numbers $n$ and $k$ such that $n^k-n$ is not a multiple of $k$. One such example is $n=2$ and $k=4$: then $n^k-n=16-2=14$, which is not a multiple of 4.

8. (d) The negation of the statement is $\forall x\in\mathbb{Z}\exists n\in\mathbb{Z}, x=n+2$. The negation is true. Here is a proof: let $x\in\mathbb{Z}$ be any integer. Then choose $n=x-2$. Note that $n$ is also an integer, and moreover, $n+2=(x-2)+2=x$. This proves the negated statement.

(e) The negation of the statement is $\exists y\in\{x|x\in\mathbb{Z},x\geq 1\}, 5y^2+5y+1$ is not a prime number. The negation is again true. To prove it we need to give an integer $y$ which is greater than or equal to 1 and such that $5y^2+5y+1$ is not a prime number. An example is $y=12$, in that case $5y^2+5y+1=781=11*71$.

(f) The negation of the statement is $\exists y\in\{x|x\in\mathbb{Z},x^2<0\}, 5y^2+5y+1$ is not a prime number. In this case, the original statement is true, because $\{x|x\in\mathbb{Z},x^2<0\}=\emptyset$, and any statement of the form $\forall x\in\emptyset, P(x)$ is true for trivial reasons.

9. We are given that 8881 is not a prime, so there are integers $a$ and $b$, both greater than 1, such that $8881=ab$. Choose such a factorization with $a$ as small as possible. Then $a$ has to be a prime (otherwise any factor of $a$ would be a smaller factor for 8881). Also, $a\leq b$, so $8881=ab\geq a^2$, so $a\leq\sqrt{8881}$. Since

$97^2 > 8881$, this implies that $a < 97$. The largest prime below 97 is 89, so $a \leq 89$.