## MAT 1341, INTRODUCTION TO LINEAR ALGEBRA, WINTER 2003 Midterm, March 7 Prof. P. Selinger

## FAMILY NAME: \_\_\_\_\_\_ FIRST NAME: \_\_\_\_\_ ID: \_\_\_\_\_

Question:	1	2	3	4	5	6	7	8	Total
Possible Points	2	2	2	2	3	3	4	4	22
Actual Points:									

## PLEASE READ THESE INSTRUCTIONS CAREFULLY.

- 1. You have 80 minutes to complete this test.
- 2. This is a closed book test, and no notes of any kind are allowed. The use of calculators is neither required nor permitted.
- 3. The test has 8 questions.

Questions 1–6 are multiple choice or short answer questions. Make sure you check your answer carefully, as there will be no part marks given.

Questions 7–8 require a detailed answer. Please write legibly and reason carefully. You can write on the back of pages if necessary.

**Question 1.** Under what condition can a point (a, b, c) be written as a linear combination of (1, 2, 0) and (1, 1, 1)?

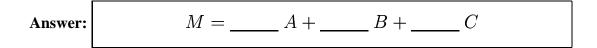
- $\Box \quad 3a b 2c = 0.$  $\Box \quad a + b 2c = 0.$  $\Box \quad 2a b c = 0.$  $\Box \quad 2a b + 2c = 0.$  $\Box \quad a b = 0.$
- $\Box \quad a 3b + 2c = 0.$

Question 2. Suppose that a given matrix A satisfies  $A^2 - 2A - I = 0$ . Give a formula for  $A^{-1}$ :

- $\Box \quad A^{-1} = 2A + I.$
- $\Box \quad A^{-1} = A + I.$
- $\Box \quad A^{-1} = A 2I.$
- $\Box \quad A^{-1} = 2A I.$
- $\Box \quad A^{-1} = A + 2I.$
- $\Box \quad A^{-1} = A I.$

Question 3. In the vector space  $M_{2,2}$  of real  $2 \times 2$ -matrices, express M as a linear combination of the matrices A, B, and C, where

$$M = \begin{pmatrix} 4 & 7 \\ 7 & 9 \end{pmatrix}, \qquad A = \begin{pmatrix} 1 & 1 \\ 4 & 5 \end{pmatrix}, \qquad B = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, \qquad C = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix},$$



**Question 4.** Write the polynomial  $p(t) = t^2 - 2t + 1$  as a linear combination of the three polynomials  $q_1(t) = t^2 - 1$ ,  $q_2(t) = t^2 + t$  and  $q_3(t) = t^2 + t + 1$ .

Answer: 
$$p = \_ q_1 + \_ q_2 + \_ q_3$$

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**Question 5.** Determine whether or not W is a subspace of  $\mathbb{R}^3$  where W consists of all vectors (a, b, c) in  $\mathbb{R}^3$  such that:

(a)	$b = a^2$	□ yes	🗆 no
(b)	a = 2b = 3c	□ yes	🗌 no
(c)	a = 3b	□ yes	🗌 no
(d)	ab = 0	□ yes	🗆 no
(e)	$a \leqslant b \leqslant c$	□ yes	🗆 no
(f)	a + b + c = 0	□ yes	🗆 no

**Question 6.** Let V be a vector space over a field K. Which of the following statements are always valid:

(a) Every subset of $V$ is a subspace of $V$	□ true	☐ false
(b) Every subspace of $V$ is a subset of $V$	□ true	□ false
(c) $\{0\}$ is a subspace of V	□ true	□ false
(d) Let $u, v \in V$ be vectors, and let $W$ be a subspace of $V$ . If $W$ then $W$ also contains the sum $u + v$ .	contains the v	ectors $u$ and $v$ , false
(e) Let $u, v \in V$ be vectors, and let $W$ be a subspace of $V$ . If $W$ $W$ also contains $u$ and $v$ .	contains the s	um $u + v$ , then $\Box$ false

**Question 7.** Let W be a subset of a vector space V over a field K.

(a) Write down the three conditions which W must satisfy in order to be a subspace of V:

(1)

(2)

(3)

(b) Show that  $W = \{(x, y, z) \in \mathbb{R}^3 \mid x + y \ge z\}$  is not a subspace of  $\mathbb{R}^3$  by showing, in a specific example, that one of the above three conditions is violated.

**Question 8.** (a) Let  $V = \mathbf{P}(t)$  be the vector space of polynomials in the variable t, with real coefficients. Let  $W = \{p(t) \in V \mid p(2) = p(-2)\}$ . Show that W is a subspace of V.

(b) Recall that an  $n \times n$ -matrix A is called *symmetric* if  $A = A^T$ . Prove that the set of all symmetric  $n \times n$ -matrices is a subspace of  $\mathbf{M}_{n,n}$ .