MAT 1341, INTRODUCTION TO LINEAR ALGEBRA, WINTER 2003

Answers to Minitest 1 (Version 1)

Question 1. For which values of a and b does the system

$$\begin{cases} -x + 3y + 2z = -8 \\ x + z = 2 \\ 2x + 2y + az = b \end{cases}$$

have more than one solution?

A. if $a = -4$ and $b \neq 0$.

B. if $a \neq -4$ and $b \neq 0$.

C. $\boxed{\text{if } a = 4 \text{ and } b = 0.}$

D. if $a \neq 4$ and $b \neq 0$.

E. if $a = 4$ and $b \neq 0$.

F. if $a = -4$ and $b = 0$.

Question 2. Let $A = \begin{pmatrix} 3 & 1 & 0 \\ 2 & 3 & -1 \\ 0 & 2 & -1 \end{pmatrix}$. Then the main diagonal of A^{-1} is:

A. $\boxed{1, 3, -7.}$

B. $-1, -3, -6.$

C. $1, -3, -7.$

D. $-1, 3, -6.$

E. $-1, -3, -7.$

F. $1, 3, -6.$

Question 3. For which values of a does the matrix $\begin{pmatrix} 1 & -a & 2 \\ 0 & 1 & -2 \\ 2 & 1 & a \end{pmatrix}$ have rank 2?

A. $a = -3/2$ and $a = 1$.

B. $\boxed{a = 2/5.}$

C. No value of a.

D. $a = 3/4$ and $a = -1/2$.

E. $a = -4/3$.

F. $a = 3/4$.

Question 4. Given a non-homogeneous system of 5 equations in 7 unknowns, answer by yes or no the following three questions and indicate which combination of answers is right.

- Can the system have no solution?
- Can the system have infinitely many solutions?
- Can the system have a unique solution?

A. No, No, No.

B. Yes, Yes, Yes.

C. No, No, Yes.

D. $\boxed{\text{Yes, Yes, No.}}$

E. No, Yes, Yes.

F. Yes, No, Yes.

Questions 5 and 6 are short answer questions.

Question 5 (3 points). Find scalars $a, b, c \in \mathbb{R}$ such that $au_1 + bu_2 + cu_3 = w$, where

$$u_1 = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}, \quad u_2 = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}, \quad u_3 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \quad w = \begin{pmatrix} 3 \\ -1 \\ 3 \end{pmatrix}.$$

Answer: $a = \boxed{-1}$ $b = \boxed{1}$ $c = \boxed{2}$

Question 6 (3 points). Row reduce the following matrix to row canonical form:

$$\begin{bmatrix} 1 & 1 & 1 & -3 & 2 \\ 1 & 2 & 0 & -4 & -1 \\ 2 & 1 & 3 & -5 & 0 \end{bmatrix}.$$

Answer: $\begin{bmatrix} 1 & 0 & 2 & -2 & 0 \\ 0 & 1 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

Questions 7 and 8 require a detailed answer. Show all your work. You can use the backs of pages if necessary.

Question 7 (3 points). Find a matrix A such that $AB = C$, where

$$B = \begin{pmatrix} 1 & -1 \\ 2 & 2 \end{pmatrix}, \quad C = \begin{pmatrix} 8 & 4 \\ -3 & -1 \end{pmatrix}.$$

Answer: We first find B^{-1}:

$$\left[\begin{array}{cc|cc} 1 & -1 & 1 & 0 \\ 2 & 2 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{cc|cc} 4 & 0 & 2 & 1 \\ 0 & 4 & -2 & 1 \end{array}\right] \sim \left[\begin{array}{cc|cc} 1 & 0 & 1/2 & 1/4 \\ 0 & 1 & -1/2 & 1/4 \end{array}\right]$$

so

$$B^{-1} = \frac{1}{4}\begin{pmatrix} 2 & 1 \\ -2 & 1 \end{pmatrix}.$$

Then

$$A = CB^{-1} = \frac{1}{4}\begin{pmatrix} 8 & 4 \\ -3 & -1 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ -2 & 1 \end{pmatrix} = \frac{1}{4}\begin{pmatrix} 8 & 12 \\ -4 & -4 \end{pmatrix} = \begin{pmatrix} 2 & 3 \\ -1 & -1 \end{pmatrix}.$$

Question 8 (3 points). Consider the homogeneous system $Ax = 0$, where

$$A = \begin{pmatrix} -1 & 2 & 2 & 3 \\ 2 & -1 & 2 & 0 \\ 2 & 0 & 4 & 2 \end{pmatrix}.$$

Find a basis for the solution space of this system.

Answer:

$$A = \begin{bmatrix} -1 & 2 & 2 & 3 \\ 2 & -1 & 2 & 0 \\ 2 & 0 & 4 & 2 \end{bmatrix} \sim \begin{bmatrix} -1 & 2 & 2 & 3 \\ 0 & 3 & 6 & 6 \\ 0 & 4 & 8 & 8 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & -2 & -3 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

So the general solution is:

$$\begin{aligned} x_4 &= a \\ x_3 &= b \\ x_2 &= -2a - 2b \\ x_1 &= -a - 2b, \end{aligned} \qquad x = \begin{pmatrix} -a - 2b \\ -2a - 2b \\ b \\ a \end{pmatrix} = a\begin{pmatrix} -1 \\ -2 \\ 0 \\ 1 \end{pmatrix} + b\begin{pmatrix} -2 \\ -2 \\ 1 \\ 0 \end{pmatrix}$$

A basis for the solution space is:

$$\left\{ \begin{pmatrix} -1 \\ -2 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} -2 \\ -2 \\ 1 \\ 0 \end{pmatrix} \right\}$$