MAT 1341, INTRODUCTION TO LINEAR ALGEBRA, WINTER 2003

Minitest 2, March 28

Prof. P. Selinger

FAMILY NAME: ________________ **FIRST NAME:** ________________ **ID:** ________________

Question:	1	2	3	4	5	6	7	8	Total
Possible Points	2	2	3	3	3	3	4	4	24
Actual Points:									

PLEASE READ THESE INSTRUCTIONS CAREFULLY.

1. You have 80 minutes to complete this test.
2. This is a closed book test, and no notes of any kind are allowed. The use of calculators is neither required nor permitted.
3. The test has 8 questions.

   Questions 1–4 are multiple choice questions. Make sure you check your answer carefully.

   Questions 5–8 require a detailed answer. Please write legibly and reason carefully. You can write on the back of pages if necessary.

Question 1 (2 points). Determine for which value(s) of t the matrix

$$\begin{pmatrix} 1 & 2 & -1 \\ 2 & 0 & t \\ 0 & 1 & 1 \end{pmatrix}$$

is invertible.

- ☐ $t \neq 3$.
- ☐ $t \neq -6$.
- ☐ $t = -1$.
- ☐ $t = -6$.
- ☐ $t = 3$.
- ☐ $t \neq -1$.

Question 2 (2 points). Find the the coordinates of the matrix $A = \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$ relative to the basis

$$\mathcal{B} = \left\{ \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \right\}$$

of $M_{2,2}$.

- ☐ $[A]_{\mathcal{B}} = [1, -1, 0, 2]$.
- ☐ $[A]_{\mathcal{B}} = [1, 1, 0, 1]$.
- ☐ $[A]_{\mathcal{B}} = [-1, 1, 1, 1]$.
- ☐ $[A]_{\mathcal{B}} = [2, 0, 0, 1]$.
- ☐ $[A]_{\mathcal{B}} = [1, -1, 2, 1]$.
- ☐ $[A]_{\mathcal{B}} = [1, 1, -1, 0]$.

Question 3 (1/2 point each). Let V be an n-dimensional vector space. True or false:

(a)	If the vectors $v_1, \dots, v_m$ span V, then $m < n$.	☐ true	☐ false
(b)	Any n vectors which span V are linearly independent.	☐ true	☐ false
(c)	Every set of n vectors in V is linearly independent.	☐ true	☐ false
(d)	V has a basis consisting of n elements.	☐ true	☐ false
(e)	V is spanned by $n-1$ or fewer vectors.	☐ true	☐ false
(f)	Any $n+1$ or more vectors in V are linearly dependent.	☐ true	☐ false

Question 4 (1/2 point each). For each of the following sets of vectors, determine whether they are linearly independent or dependent:

(a)	$(0,0,0)$, $(1,0,0)$	☐ independent	☐ dependent
(b)	$(1,0,0)$, $(1,1,0)$, $(1,1,1)$	☐ independent	☐ dependent
(c)	$(4,2,2)$, $(5,1,0)$, $(3,4,2)$, $(-1,0,9)$	☐ independent	☐ dependent
(d)	$(1,5,5)$, $(3,3,2)$	☐ independent	☐ dependent
(e)	$(1,3,4)$, $(1,0,1)$, $(0,1,2)$	☐ independent	☐ dependent
(f)	$(1,3,5)$, $(1,2,3)$, $(1,1,1)$	☐ independent	☐ dependent

Question 5 (3 points). Find a basis and the dimension of the subspace W of $\mathbb{R}^3$ where:

(a) $W = \{(a, b, c, d) \mid a + b + c + d = 0\}$.

(b) $W = \{(a, b, c, d) \mid a = 2b$ and $c = 2d\}$.

Question 6 (3 points). Find a basis and the dimension of span(u_1, u_2, u_3, u_4) in $\mathbf{P}_3(t)$, where

$$u_1 = 2t^3 + 3t^2 + 4t + 5,$$
$$u_2 = -1t^3 + 1t^2 + 3t + 0,$$
$$u_3 = 1t^3 + 2t^2 + 3t + 3,$$
$$u_4 = 2t^3 + 1t^2 + 0t + 4$$

Question 7 (4 points). Consider the following matrix:

$$B = \begin{pmatrix} 1 & 5 & 3 & 2 & -1 \\ 4 & -2 & 0 & 1 & 2 \\ 3 & -1 & 1 & 2 & 1 \\ 2 & 6 & 4 & 3 & -1 \\ 4 & 0 & 2 & 3 & 1 \end{pmatrix}$$

(a) What is the rank of B?

(b) Find a subset of the rows of B which forms a basis of the row space of B.

(c) Complete the set $u_1 = (1, 3, 1, 5, 3)$, $u_2 = (1, 2, 1, 3, 2)$, $u_3 = (1, 0, 1, 0, 0)$ to a basis of $\mathbb{R}^5$.

Question 8 (4 points). Recall that a function $F : V \to U$ is linear if (1) for all $v, w \in V$, $F(v + w) = F(v) + F(w)$, and (2) for all $v \in V$, $k \in K$, $F(kv) = kF(v)$.

(a) Show that the following function is linear: $F : \mathbb{R}^2 \to \mathbb{R}^2$, where $F(x, y) = (x + y, x)$.

(b) Show that the following function is not linear: $F : \mathbb{R}^2 \to \mathbb{R}^2$, where $F(x, y) = (xy, x)$. Do this by giving a *concrete* example where one of the above laws ((1) or (2)) is violated.