Math 2030, Matrix Theory and Linear Algebra I, Winter 2014

Practice Midterm

Part I: True or false questions

Decide whether each statement is true or false. If it is false, give a reason.

Problem 1. For any two non-zero, non-parallel vectors $\mathbf{u}, \mathbf{v}$ in $\mathbb{R}^3$, the set of linear combinations of $\mathbf{u}$ and $\mathbf{v}$ is a plane through the origin.

Problem 2. The rank of a matrix is equal to the number of its non-zero rows.

Problem 3. The vector $\mathbf{z} = \begin{bmatrix} 0 \\ -\frac{1}{2} \\ 1 \end{bmatrix}$ can be written as a linear combination of $\mathbf{u} = \begin{bmatrix} -4 \\ 2 \\ 3 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 0 \\ 3 \\ -6 \end{bmatrix}$, and $\mathbf{w} = \begin{bmatrix} 4 \\ 5 \\ 0 \end{bmatrix}$.

Problem 4. A homogeneous set of linear equations is always consistent.

Problem 5. If $\mathbf{u}, \mathbf{v}$ are non-zero, non-parallel vectors in $\mathbb{R}^3$, then $\text{proj}_{\mathbf{u}} \mathbf{v} \neq \text{proj}_{\mathbf{v}} \mathbf{u}$.

Problem 6. Let $P = (0, 0)$, $Q = (4, 2)$, and $R = (1, 3)$. The point on the line $\overline{PQ}$ that is closest to R is $(2, 1)$.

Problem 7. A homogeneous system of 5 linear equations in 7 variables over the real numbers always has an infinite number of solutions.

Problem 8. A plane in $\mathbb{R}^3$ containing $\begin{bmatrix} -1 \\ 2 \\ 4 \end{bmatrix}$ and with normal vector $\begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$ has has general equation $2x - y + 3z = 5$.

Part II: Multiple choice questions

Problem 9. If $\mathbf{w} = \begin{bmatrix} 4 \\ 2 \\ r \end{bmatrix}$ is a linear combination of $\mathbf{u} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, then r must be:

(a) 1 (b) 3 (c) 0 (d) 2

Problem 10. The vectors $\begin{bmatrix} 1 \\ 3 \\ z \end{bmatrix}$ and $\begin{bmatrix} x \\ -6 \\ 2 \end{bmatrix}$ are collinear (parallel) if:

(a) $x = 2, z = 1$ (b) $x = -2, z = -1$ (c) $x = -1, z = 2$ (d) $x = 1, z = 2$

Problem 11. The system over $\mathbb{Z}_3$

$$\begin{array}{ccccccc} x & + & y & & & = & 2 \\ x & + & 2y & + & 2z & = & 0 \\ & & 2y & + & z & = & 2 \end{array}$$

has

(a) no solution (b) one solution (c) 3 solutions (d) an infinite number of solutions

Problem 12. For all non-zero vectors $\mathbf{v}$ in $\mathbb{R}^n$, the non-zero vector $\mathbf{u}$ is orthogonal to:

(a) $\text{proj}_{\mathbf{u}}(\mathbf{v})$ (b) $\text{proj}_{\mathbf{v}}(\mathbf{u})$ (c) $\mathbf{v} + \text{proj}_{\mathbf{u}}(\mathbf{v})$ (d) $\mathbf{v} - \text{proj}_{\mathbf{u}}(\mathbf{v})$

Part III: Detailed answer questions

Problem 13. If $\mathbf{p} = [-1, 4, 3]$, $\mathbf{q} = [1, -2, -2]$, find a unit vector perpendicular to both $\mathbf{p}$ and $\mathbf{q}$.

Problem 14. Find the line of intersection (in vector form) of the following two planes:

$$-x + 2y - z + 1 = 0, \quad y + 3z - 1 = 0.$$

Problem 15. Find the distance from the point $Q = (2, 2, 1)$ to the plane $x + y - z = 0$.