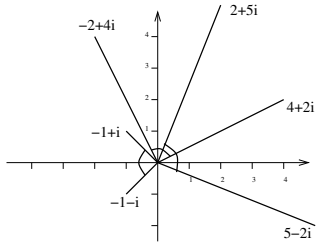
MAT 3321, COMPLEX ANALYSIS AND INTEGRAL TRANSFORMS, WINTER 2005

Answers to Homework 1
12.1 #2,4,10; 12.2 #2

Problem 12.1 #2 The graph on the right shows $z = 4+2i$ and $iz = -2+4i$. It can be seen that the angle of rotation is indeed $90°$. Also shown are $z = -1+i$ and $iz = -1-i$, as well as $z = 5-2i$ and $iz = 2+5i$. In each case, the angle of rotation can be seen to be $90°$.



Problem 12.1 #4 Let $z_1 = 4+3i$ and $z_2 = 2-5i$. Then

$$\begin{aligned}(3z_1 - z_2)^2 &= (3(4+3i) - (2-5i))^2 \\ &= (12+9i-2+5i)^2 \\ &= (10+14i)^2 \\ &= 100+280i+196i^2 \\ &= 100+280i-196 \\ &= -96+280i.\end{aligned}$$

Problem 12.1 #10 Let $z_1 = 4+3i$.

$$1/z_1^2 = 1/(4+3i)^2 = 1/(16+24i-9) = 1/(7+24i) = \frac{7-24i}{(7+24i)(7-24i)}$$

$$= \frac{7-24i}{49+576} = \frac{1}{625}(7-24i) = \frac{7}{625} - \frac{24}{625}i$$

Also,

$$1/\bar{z}_1^2 = 1/\overline{z_1^2} = \overline{1/z_1^2} = \frac{7}{625} + \frac{24}{625}i$$

Problem 12.2 #2 Let $z = -2+2i$. We have $z = re^{i\theta}$ where $r = |z| = \sqrt{4+4} = \sqrt{8}$ and $\theta = \mathrm{Arg}(z) = \arctan(2/-2) + \pi = 3\pi/4$. Thus $z = \sqrt{8}e^{\frac{3pi}{4}i} = 2.828e^{2.356i}$.