2010 Category Theory Octoberfest

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Abstracts

Steve Awodey (CMU): "Sketch of the homotopy interpretation of intensional type theory"

Abstract: As a tutorial of sorts, I will outline the homotopy interpretation of intensional type theory and survey some of the recent results by various people.

Nathan Bowler (Cambridge): "Unwirings and exponentiability for multicategories"

Abstract: I'll introduce the concept of an unwiring (roughly, a way of pulling things apart over the arities given by a monad, just as the structure map of an algebra gives you a way of pasting them together over those arities), and explain how it arises in the study of categories of games. I'll explain the close links of this concept to exponentiability, including a characterisation of exponentiable multicategories in well-behaved contexts.

Robin Cockett (Calgary): "Integral categories"

Abstract: An integral category is a cartesian left additive category with an integral operator:

$$\frac{A \otimes A \longrightarrow B; (a,v) \mapsto f(a) \cdot v}{A \otimes A \longrightarrow B; (a,v) \mapsto \left[\int_{a_0}^{v_0} f(a_0).v_0\right](a,v)}$$

which takes a map which is additive in the second argument, a form, and produces a function of the same type (but which is, in general, no longer additive in the second argument). Integral categories provide an abstraction of integration on forms, rather than measure theory. Integration in these categories is treated a binding operator on forms which is first explored independently of differentiation. When an integral category is also a (cartesian) differential category it is reasonable to link the two structures by the two fundamental theorems of calculus. One then obtains a setting in which the relation between integration on forms and differentiation is formally the same as in calculus.

Geoff Cruttwell (Calgary): "Differential and tangent structure for restriction categories"

Abstract: Differential restriction categories capture partial settings in which one

can differentiate maps. However, their axiomatization is not closed to a number of important constructions. For example, the category of manifolds constructed from a differential restriction category is not itself a differential restriction category. This is an obstruction to doing significant differential/algebraic geometry in these settings.

In this talk we will show that by considering tangent bundles, one can obtain an alternative description of differential structure which is transportable along these constructions.

Jonathan Gallagher (Calgary): "Differential Join Restriction Categories"

Abstract: A differential restriction category is a restriction category which has a combinator which takes a map to its derivative. A join restriction category is a restriction category where any two compatible maps have a join with respect to the restriction partial ordering. This means that the "open set lattices" in join restriction categories are distributive lattices. The combination of differential and join restriction structure is a basis for further constructions. These basic structures will be the subject of this talk and an interesting class of examples from algebraic geometry will be provided.

Alex Hoffnung (Ottawa): "Groupoidification and the Hecke Bicategory"

Abstract: The groupoidification program initiated by Baez, Dolan, Trimble, Walker and Hoffnung has found success in applications to mathematical physics and in understanding certain structures from representation theory. The categorification of Hecke algebras in work of Baez and Hoffnung is a first example. Forthcoming work of Baez and Walker on Hall algebras will further illuminate the robustness of the tools of groupoidification. I will speak about the role of enriched bicategories and bicategories of spans in constructing the Hecke bicategory of a finite group G, a categorification of the category of permutation representations of G. As an immediate corollary and time permitting, when G is the simple group attached to a Dynkin diagram D and a prime power q, we will see a categorification of the usual Hecke algebra H(D,q).

André Joyal (UQAM): "The Koszul sign rule in Feynman diagrams"

Abstract: It is well known that the category of graded modules is symmetric monoidal, if the symmetry is given by the Koszul sign rule. It is also well known that Feynman diagrams can be used in any symmetric monoidal category. Odd degree morphisms are frequently used in algebraic topology and homological algebra. But the category of graded R-modules stops been symmetric monoidal if these morphisms are included into the category. We introduce a notion of oriented Feynman diagram and explain how these diagrams can be used for computing with odd degree morphisms as well. We will illustrate with a few examples taken from Koszul duality.

Chris Kapulkin (Pittsburgh): "Π- and Σ-types in homotopy theoretic models of type theory"

Abstract: Connections between Intensional Martin-Löf Type Theory and homotopy theory have been recently intensively studied, see for example: [1,3,2]. By interpreting type theory in Quillen model categories—or more generally, in categories equipped with a (natural) weak factorization system—we obtain a wide class of models for a type theory being an extension of a theory with only one type constructor Id. In such an interpretation we allow only fibrant objects in a category to be interpretations of types.

In my talk, I would like to show how to fit other type constructors: Π and Σ into this interpretation. One may observe that Σ-types always exist (since their interpretation is given by composition). The existence of Π-types is more complicated and one has to impose some additional conditions on a model category $\mathbb{C}$, namely:

1. $\mathbb{C}$ is right proper.
2. cofibrations in $\mathbb{C}$ are stable under pullback.
3. for any fibration f in $\mathbb{C}$ there exists a right adjoint to the pullback functor along f.

These conditions suffice for $\mathbb{C}$ to be a model for Π-types and lead to a numerous of examples such as: the category **Gpd** of groupoids, the category **PreOrd** of preorders, the category $\mathbf{Sets}$ of simplicial sets, and other interesting examples coming from algebraic topology and algebraic geometry.

[1] S. Awodey and M. A. Warren. Homotopy theoretic models of identity types. *Math. Proc. of the Cam. Phil. Soc.*, 2009.

[2] R. Garner and B. van den Berg. Topological and simplicial models of identity types. Submitted, 2010.

[3] M. A. Warren. *Homotopy Theoretic Aspects of Constructive Type Theory*. PhD thesis, Carnegie Mellon University, 2008.

Toby Kenney (Dalhousie): "Generalised Sup Arrows and the Totally Below Relation"

Abstract: The category of complete lattices and functions which preserve all suprema is an important category. There is also an important relation, known as the totally below relation << on any complete lattice. It is defined by x << y if for any downset S whose supremum is greater than or equal to y, x must be a member of S. As observed by Raney, a lattice is completely distributive if and only if every element is the supremum of the set of elements totally below it. This was used by Fawcett & Wood, and Rosebrugh & Wood to provide and study a constructive definition of completely distributive lattices. We extend this concept to arbitrary ordered sets, and obtain a concept analogous to complete distributivity, but without requiring the ordered set to be a lattice.

The totally below relation is not in general reflexive, but is interpolative and transitive. We look at how we can study an ordered set by looking at its totally below relation as an interpolative and transitive relation. (Joint with R. J. Wood)

Aleks Kissinger (Oxford): “Frobenius States and a Graphical Language for Multipartite Entanglement”

Abstract: While multipartite quantum states constitute a key resource for quantum computations and protocols, obtaining a high-level, structural understanding of entanglement involving arbitrarily many qubits is a long-standing open problem in quantum computer science. We approach this problem by identifying the close connection between a special class of highly entangled tripartite states, which we call Frobenius states, and the standard notion of a commutative Frobenius algebra. We use these states (and their induced algebras) as a primitive in a graphical language that is both universal for quantum computation and capable of highlighting the behavioural differences in types of Frobenius states. This graphical language then suggests methods for state preparation, classification, and efficient classical modeling of certain kinds of quantum systems.

Fred Linton (Wesleyan): “How to see the reals as compact Hausdorff space, and why you’d want to”

Abstract: In the course of his book Stone Spaces, Peter Johnstone seemingly has occasion to lament the real line’s not being a compact Hausdorff space:

The real Gelfand Duality adjoint equivalence KT_2 ~ C*-Alg, while of the form Hom(-, R) in either direction, is so without R being a "schizophrenic" object (while it has a personality in C*-Alg, it has none in KT_2, not being a compact space).

This lament misses the point: the underlying personality of the real line R (or for that matter, the complex plane C) as 1-dimensional [real | complex] Banach

algebra/*-algebra/space is its unit disc D, which most certainly *is* a compact Hausdorff space; and KT_2(X, D) is the unit disc of the C*-algebra or Banach space KT_2(X, R) (or KT_2(X, C)) involved in Gelfand Duality.

The talk proposed will riff a bit on these and related themes.

Rory Lucyshyn-Wright (York): "Totally Distributive Toposes, Continuous Categories, and Injective Toposes"

Abstract: A locally small category $\mathcal{E}$ is totally distributive (as defined by Rosebrugh-Wood) if there exists a string of adjoint functors $W \dashv X \dashv Y$, where $Y : \mathcal{E} \to \hat{\mathcal{E}}$ is the Yoneda embedding. We show that the Grothendieck topos associated to any *ind-small continuous category* (in the sense of Johnstone and Joyal) is totally distributive. Moreover, we show that if the given ind-small continuous category is also cocomplete, then the associated topos is *lex totally distributive*, meaning that the left adjoint $W : \mathcal{E} \to \hat{\mathcal{E}}$ preserves finite limits. Conversely, we show that the category of points of any lex totally distributive Grothendieck topos is an ind-small continuous category.

Thus, by Johnstone-Joyal we deduce the following corollaries to these results. Firstly, every *quasi-injective* Grothendieck topos (i.e. any retract of a presheaf topos by geometric morphisms) is totally distributive. Secondly, every *injective* Grothendieck topos (equivalently, any retract of a presheaf topos on a finitely complete site) is lex totally distributive. Lastly, any lex totally distributive Grothendieck topos is quasi-injective.

In view of a result of Walters on *lex total* categories, a totally distributive category is a Grothendieck topos as soon as it has a small generator, whence we obtain further corollaries to the above.

These results constitute a partial generalization of the classical duality between continuous dcpos and completely distributive lattices established by J. Lawson and R.-E. Hoffmann.

Gábor Lukács (Manitoba): "Solving problems in topological groups (and number theory) using category theory"

Abstract: If H is a countable subgroup of the group $\mathbb{T} := \mathbb{R}/\mathbb{Z}$, then there is a sequence $\{n_k\}_{k=1}^{\infty}$ of integers such that $H = \{x \in \mathbb{T} \mid \lim\limits_{k\to\infty} n_k x = 0\}$ (cf. [2]). This result can be generalized if one replaces $\mathbb{T}$ with an arbitrary compact abelian group C, and the integers with the group $\widehat{C}$ of continuous characters of C (cf. [1], [5], and [6]). The first two proofs are based on complicated approximations; the third one is based on translating the problem into the language of category theory,

and interpreting it as a question about a bicoreflective subcategory—leading to a 6-line proof. The aim of this talk is to expose the categorical machinery underpinning [6].

Precompact topological groups are subgroups of compact ones. It follows from the Comfort-Ross duality (cf. [3]) that one can view a precompact abelian group as a pair (A, H) of an abelian group A and a subgroup H of the (compact Hausdorff) abelian group $\widehat{A} := \hom(A, \mathbb{T})$. Every idempotent closure operator c defined on (pre)compact Hausdorff abelian groups gives rise to a bicoreflection $(A, H) \longmapsto (A, c_{\widehat{A}}(H))$. In this talk, the relationship between closure operators and the associated bicoreflections will be discussed.

[1] M. Beiglböck, C. Steineder, and R. Winkler. Sequences and filters of characters characterizing subgroups of compact abelian groups. *Topology Appl.*, 153(11):1682–1695, 2006.

[2] A. Bíró, J.-M. Deshouillers, and V. T. Sós. Good approximation and characterization of subgroups of $\mathbb{R}/\mathbb{Z}$. *Studia Sci. Math. Hungar.*, 38:97–113, 2001.

[3] W. W. Comfort and K. A. Ross. Topologies induced by groups of characters. *Fund. Math.*, 55:283–291, 1964.

[4] D. Dikranjan. Closure operators in topological groups related to von Neumann's kernel. *Topology Appl.*, 153(11):1930–1955, 2006.

[5] D. Dikranjan and K. Kunen. Characterizing subgroups of compact abelian groups. *J. Pure Appl. Algebra*, 208:285–291, 2007.

[6] G. Lukács. Precompact abelian groups and topological annihilators. *J. Pure Appl. Algebra*, 208(3):1159–1168, 2007.

Peter LeFanu Lumsdaine (Dalhousie): "Conservativity principles in dependent type theory: a homotopy-theoretic approach"

Abstract: In predicate logic, an interpretation of a theory T in another theory S is *conservative* if whenever S proves a theorem that could be stated in T, then T already proves it.

Generalising this along the Curry-Howard correspondence, to give a notion of conservativity between dependent type theories, is interesting and subtle. I will discuss some existing generalisations, re-state them from a homotopy-theoretic perspective, and prove a sample conservativity result using this approach.

Michael Makkai (McGill): "Semi-Strict Omega Categories"

Abstract: An attempt is made to generalize parts of the theory of Gray categories

([4], [5], [7]) to arbitrary finite dimensions (Gray categories being the case of 3 dimensions), amounting to a single ω- (or: ∞-) categorical context. The project is inspired by A. E. Crans's ideas and attempts at defining n-*teisi* (see [5] and other papers by Crans); for this reason, I talk about *Crans-categories*. The definition of Crans-(ω-)category is stated in an "unpacked style", one that presents the concept directly as one that is monadic over ω-graphs. The definition is in an abstract form that assumes (unfortunately!) the truth of certain precisely stated, but as yet unproved, conjectures, each of the form "for all n, $P(n)$" where $P(n)$ is accessible to direct verification for each finite dimension n, the universally quantified statement remaining (of course) to be the problem. Indeed, the conjectures have been verified for a considerable number of cases. The aim is to arrive at a concept that is equivalent, in the sense of an appropriate notion of equivalence, to the, at the present, hypothetical concept of "completely" weak ω-category, one which gives "completely" weak ω-categories as algebras of a monad over ω-graphs. In other words, the ultimate goal is to prove an ω-dimensional generalization of the main result, for dimension 3, of [4] (see also [7]). It is conjectured that Crans-ω-categories, defined as indicated in an "unpacked" manner, turn out to be the same as ones enriched over CRANS, the closed multicategory of all small Crans-ω-categories. To deal with enrichment, I follow [2], and [3] in using the concept of closed multicategory, rather than the original concept of closed category in [1]. I make no use of tensor products, which makes the work dissimilar to [5].

[1] S. Eilenberg and G. M. Kelly, Closed Categories. In: La Jolla 1965 Proceedings; Springer 1966; pp. 421-562.

[2] P. E. J. Linton, The Multilinear Yoneda Lemmas. In: Reports of the Midwest Category Seminar V; SLNM 195, 1971; pp. 209-229.

[3] J. Lambek, Multicategories Revisited. In: AMS Contemporary Mathematics, Vol. 92, 1989; pp. 217-239.

[4] R. Gordon, A. J. Power and R. Street, Coherence for Tricategories. Memoirs of the AMS, Volume 117, No. 558, 1995.

[5] A. E. Crans, A Tensor Product for Gray-Categories. TAC 5(1999), pp. 12-69.

[6] M. Makkai, The Word Problem for Computads. 2005. Manuscript, 146 pages. At: www.math.mcgill.ca/makkai

[7] Nick Gurski, An Algebraic Theory of Tricategories. Ph. D thesis, 2007. 164 pages. At: the author's website.

Susan Niefield (Union): "A Double Category of Topological Spaces"

Abstract: Let $\mathbb{T}\mathrm{op}_2 \overrightarrow{\underrightarrow{\rightarrow}} \mathbb{T}\mathrm{op}_1 \overrightarrow{\underleftarrow{\rightarrow}} \mathbb{T}\mathrm{op}_0$ denote the double category whose objects are topological spaces X, horizontal morphisms $f: X \longrightarrow Y$ are continuous maps, vertical morphisms $l: X_0 \bullet\!\!\longrightarrow X_1$ are finite intersection preserving maps $l: \mathcal{O}(X_0) \longrightarrow \mathcal{O}(X_1)$ on the open set lattices, and cells are of the form

$$\begin{tikzcd} X_0 \arrow[r, "f_0"] \arrow[d, "l"', "\bullet" marking] \arrow[dr, phantom, "\supseteq"] & Y_0 \arrow[d, "m", "\bullet" marking] \\ X_1 \arrow[r, "f_1"'] & Y_1 \end{tikzcd}$$

This talk we will consist of two unrelated parts. First, we will consider the relationship between $\mathbb{T}\mathrm{op}$ and the cospan double category $\mathbb{C}\mathrm{osp}(\mathrm{Top})$. Then we will establish properties of $\mathbb{T}\mathrm{op}_1$ derived from its equivalence with the slice category $\mathrm{Top}/\mathbb{2}$ of topological spaces over the Sierpinski space.

Michael A. Warren (Dalhousie): "Combinatorial realizability models of type theory"

Abstract: We present a model construction for dependent type theory which allows one to prove results about the syntactic behavior of the theory, while at the same time working in a convenient semantic setting. We refer to these models as combinatorial realizability models since the approach taken is motivated by ordinary realizability models, but with certain combinatorial data associated with the syntax of the theories in question as realizers instead of computable functions. For example, in our motivating example realizers of terms of basic type will be edges in a suitable graph. This is joint work with Pieter Hofstra.