MATH 1115, Mathematics for Commerce
WINTER 2011
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Sample Final Examination Model Solutions

Calculators are not permitted.

Each multiple choice question is worth one mark, other questions are worth two marks. Include all your working for the other questions, but for multiple-choice questions, it is sufficient to just indicate the letter. [Make it clear which letter you mean as your answer. Ambiguous answers will be marked wrong.]

1. *It is impossible to pay off a loan for $20,000 at interest rate 6% compounded monthly, if the monthly payments are less than:*

   solution 1:

   The number of payments, n and amount of payments R satisfy $20000 = R\frac{1-1.005^{-n}}{0.005}$. Therefore $1 - 1.005^{-n} = \frac{20000\times 0.005}{R}$, so $1.005^{-n} = 1 - \frac{20000\times 0.005}{R}$. However, $1.005^{-n} > 0$, so repayment is impossible if $\frac{20000\times 0.005}{R} > 1$, which happens if $R < 20000 \times 0.005$.

   solution 2:

   The monthly interest is given by 20000×0.005. In order to pay off the loan, the regular payments must not be less than this interest.

   (A) 20000×1.005

   (B) $\mathbf{20000 \times 0.005}$

   (C) 20000×0.06

   (D) $\frac{20000}{1.005}$

   (E) Any monthly payments will eventually pay off the loan.

2. *A company is considering investing $100,000 in a new project. They estimate that the project will lead to the following net cash flows at the end of each year:*

End of Year	*Cash flow*
1	*$20,000*
2	*$50,000*
3	*$30,000*
4	*$30,000*

   The net present value of the project at 10% interest compounded annually is given by:

   (A) $20000 + 50000 \times 1.1^{-1} + 30000 \times 1.1^{-2} + 30000 \times 1.1^{-3}$

   (B) $20000 \times 1.1 + 50000 \times 1.1^{2} + 30000 \times 1.1^{3} + 30000 \times 1.1^{4} - 100000$

(C) $20000 \times 1.1^{-1} + 50000 \times 1.1^{-2} + 30000 \times 1.1^{-3} + 30000 \times 1.1^{-4}$

(D) $\mathbf{20000 \times 1.1^{-1} + 50000 \times 1.1^{-2} + 30000 \times 1.1^{-3} + 30000 \times 1.1^{-4} - 100000}$

(E) It is not possible to calculate the net present value from the information given.

3. *The matrix* $A = \begin{pmatrix} 1 & 2 & -3 & 3 & -2 & 4 & 0 \\ 1 & 3 & 1 & 3 & -1 & 8 & -2 \\ -1 & -3 & 0 & -1 & -3 & -6 & 5 \\ 2 & 6 & 2 & 5 & -2 & 17 & -2 \\ 2 & 5 & -2 & 6 & -4 & 14 & 0 \\ -1 & -3 & -1 & -3 & 1 & -7 & 1 \\ 1 & 4 & 5 & 2 & 0 & 13 & -1 \end{pmatrix}$ *is invertible with inverse* $A^{-1} = \begin{pmatrix} 113 & 60 & 11 & -37 & -48 & 53 & 62 \\ -44 & -26 & -4 & 18 & 17 & -22 & -26 \\ 9 & 5 & 1 & -3 & -4 & 4 & 5 \\ 3 & 3 & 0 & -4 & 0 & 1 & 3 \\ 5 & 3 & 0 & -4 & -1 & 2 & 4 \\ 1 & 1 & 0 & -1 & 0 & 1 & 1 \\ 1 & 0 & 0 & -1 & 0 & 0 & 1 \end{pmatrix}$.

Which of the following is part of the solution to the system of equations

$$\begin{array}{rrrrrrrcr}
a & +2b & -3c & +3d & -2e & +4f & & = & 2 \\
a & +3b & +c & +3d & -e & +8f & -2g & = & 3 \\
-a & -3b & & -d & -3e & -6f & +5g & = & 3 \\
2a & -4b & +2c & +5d & -2e & +17f & -2g & = & 2 \\
2a & +6b & -2c & +6d & -4e & +14f & & = & -1 \\
-a & -3b & -c & -3d & +e & -7f & +g & = & -1 \\
a & +4b & +5c & +2d & & +13f & -g & = & -2
\end{array}$$

We can solve the system of equations by multiplying the vector $\begin{pmatrix} 2 \\ 3 \\ 3 \\ 2 \\ -1 \\ -1 \\ -2 \end{pmatrix}$ by A^{-1}. Since we will only need to find one of the values in order to answer this question, we might as well start with the easiest values to calculate. It is easiest to calculate the bottom entries c, d, e, f, and g in this way. We calculate $c = 9 \times 2 + 5 \times 3 + 1 \times 3 - 3 \times 2 - 4 \times -1 + 4 \times -1 + 5 \times -2 = 18 + 15 + 3 - 6 + 4 - 4 - 10 = 20$, $e = 5 \times 2 + 3 \times$

$3 - 4 \times 2 - 1 \times -1 + 2 \times -1 + 4 \times -2 = 10 + 9 - 8 + 1 - 2 - 8 = 2$,
$f = 1 \times 2 + 1 \times 3 - 1 \times 2 + 1 \times -1 + 1 \times -2 = 2 + 3 - 2 - 1 - 2 = 0$

From this, we see that (E) must be the answer to the question. [For completeness, the full solution to the system of equations is $a = 236, b = -85, c = 20, d = 0, e = 2, f = 0, g = -2$.]

(A) $a = 54$

(B) $b = -35$

(C) $c = 10$

(D) $e = 0$

(E) $\mathbf{f} = \mathbf{0}$

4. *The feasible region for the system of inequalities*

$$\begin{aligned} 2x + 2y &\leqslant 8 \\ x - y &\geqslant 2 \\ x + 2y &\geqslant 1 \\ -2x + 4y &\geqslant 3 \end{aligned}$$

is:

3 times the second inequality, plus the fourth inequality give $x + y \geqslant 9$, which contradicts the first inequality.

It is also possible to see this by drawing a diagram showing the feasible regions.

(A) Bounded and $x = 3, y = 1$ is a corner.

(B) Empty

(C) Unbounded and $x = 7, y = -3$ is a corner.

(D) Bounded and $x = 7, y = -3$ is a corner.

(E) Unbounded and $x = 3, y = 1$ is a corner.

5. *While applying the simplex method to solve a linear programming problem to maximise Z, you obtain the following simplex tableau.*

x_1	x_2	x_3	s_1	s_2	s_3	s_4	Z	
1	0	-2	1	0	4	0	0	5
0	1	1	2	0	1	0	0	3
0	0	-2	-2	1	-1	0	0	8
0	0	1	3	0	2	1	0	11
0	0	3	2	0	1	0	1	15

What is the maximum value for Z?

The entries in the bottom row are all non-negative, so the current BFS is the one which maximises Z. The value of Z is the last entry in this row, i.e. 15.

(A) 10

(B) 15

(C) 20

(D) 23

(E) It is impossible to determine the maximum value for Z from the data given.

6. *A company manufactures a certain product. The products produced have a $\frac{1}{1000}$ chance of being defective. If the company produces 3 products to fill one order, what is the probability that none of the three products is defective? [Assume that seperate products are independant.]*

   Let A be the event that the first product is defective, B be the event that the second product is defective, C be the event that the third product is defective. The events are independant, so the probability that none is defective is

$$P(A^c \cap B^c \cap C^c) = P(A^c)P(B^c)P(C^c) = \left(1 - \frac{1}{1000}\right)^3 = \left(\frac{999}{1000}\right)^3$$

   (A) $\frac{997}{1000}$

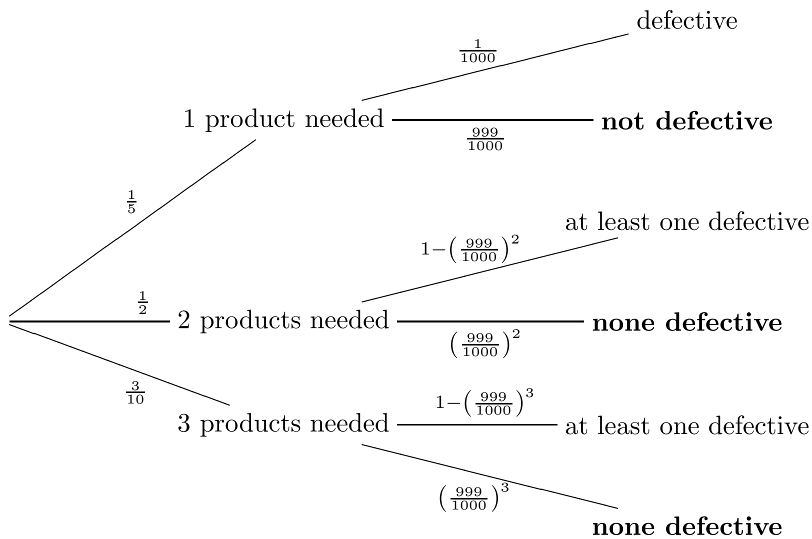
   (B) $\frac{3}{1000}$

   (C) $\boldsymbol{\left(\frac{999}{1000}\right)^3}$

   (D) $1 - \left(\frac{1}{1000}\right)^3$

   (E) $1 - \left(\frac{999}{1000}\right)^3$

7. *A second company needs some of the products produced by the company in the previous question. The probability that they need one is $\frac{1}{5}$. The probability that they need two is $\frac{1}{2}$ and the probability that they need three is $\frac{3}{10}$. What is the probability that none of the products they use is defective?*



The total probability that none is defective is therefore $\frac{1}{5}\times\frac{999}{1000}+\frac{1}{2}\left(\frac{999}{1000}\right)^2+\frac{3}{10}\left(\frac{999}{1000}\right)^3$.

(A) $\left(\frac{999}{1000}\right)^2$

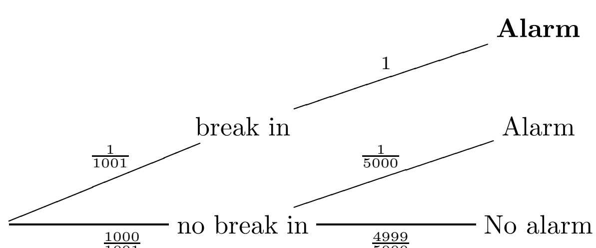
(B) $\boldsymbol{\frac{1}{5}\times\frac{999}{1000}+\frac{1}{2}\left(\frac{999}{1000}\right)^2+\frac{3}{10}\left(\frac{999}{1000}\right)^3}$

(C) $\frac{9979}{10000}$

(D) $\frac{1}{5}\left(\frac{999}{1000}\right)^3+\frac{1}{2}\left(\frac{999}{1000}\right)^2+\frac{3}{10}\times\frac{999}{1000}$

(E) 1

8. *A company has invested in a new security system, which the suppliers assure them will only give a false alarm one night in every 5,000. From looking at police statistics, the company estimates that 1 building in every 1,001 is broken into every night. From these probabilities, what is the probability that there is a break-in in the building given that the alarm goes off? [Assume the alarm always goes off if there is a break-in.]*



The total probability that the alarm goes of is $\frac{1}{1001}+\frac{1000}{1001}\times\frac{1}{5000}=\frac{6}{5005}$, while the probability that there is a break in and the alarm goes off is

$\frac{1}{1001}$. The conditional probability that there is a break in given that the alarm goes off is therefore

$$\frac{\left(\frac{1}{1001}\right)}{\left(\frac{6}{5005}\right)} = \frac{5}{6}$$

(A) $\frac{1}{6}$

(B) $\frac{2}{3}$

(C) $\frac{4}{5}$

(D) $\mathbf{\frac{5}{6}}$

(E) $\frac{300}{1001}$

9. *two fair (6-sided) dice are rolled. The probability that the higher roll is 4 (or both rolls are 4) is:*

   The rolls where the higher roll is 4 are: (1,4), (2,4), (3,4), (4,1), (4,2), (4,3) and (4,4). There are 7 such rolls out of 36 possible, equally likely rolls, so the probability is $\frac{7}{36}$.

   (A) $\frac{5}{36}$

   (B) $\frac{1}{6}$

   (C) $\mathbf{\frac{7}{36}}$

   (D) $\frac{8}{36}$

   (E) $\frac{4}{11}$

10. *Which of the following statments about limits are true:*

    (i) If $\lim_{x\to a^+} f(x)$ and $\lim_{x\to a^-} f(x)$ both exist, then so does $\lim_{x\to a} f(x)$.

    This is false — if $\lim_{x\to a^+} f(x)$ and $\lim_{x\to a^-} f(x)$ are different, then $\lim_{x\to a} f(x)$ does not exist.

    (ii) If $\lim_{x\to a} f(x)$ exists, then so do $\lim_{x\to a^+} f(x)$ and $\lim_{x\to a^-} f(x)$.

    This is true. Both $\lim_{x\to a^+} f(x)$ and $\lim_{x\to a^-} f(x)$ must be equal to $\lim_{x\to a} f(x)$.

    (iii) If $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ both exist, then so does $\lim_{x\to a} f(x)+g(x)$.

    This is true.

    $$\lim_{x\to a} f(x) + g(x) = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$$

    (iv) If $\lim_{x\to a} f(x)+g(x)$ exists, then so do $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$.

    This is false. For example, let $f(x) = \frac{1}{x-a}$ and $g(x) = -\frac{1}{x-a}$, then $f(x) + g(x) = 0$, so $\lim_{x\to a} f(x) + g(x)$ exists, but $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ do not.

    (A) Just (iv)

    (B) (i) and (iii)

(C) (ii) and (iv)

(D) (ii) and (iii)

(E) (i), (ii) and (iii)

11. *Calculate* $\lim_{x\to 1} \frac{x-1}{(x+2)(x-1)}$

When $x \neq 1$, $\frac{x-1}{(x+2)(x-1)} = \frac{1}{x+2}$, so $\lim_{x\to 1} \frac{x-1}{(x+2)(x-1)} = \lim_{x\to 1} \frac{1}{(x+2)} = \frac{1}{3}$.

(A) 0

(B) $\mathbf{\frac{1}{3}}$

(C) $\frac{1}{2}$

(D) 3

(E) ∞

12. *Calculate* $\lim_{x\to\infty} \frac{x+3}{x^2+7}$

Since the degree of the denominator is larger than the degree of the numerator, the limit is 0.

More formally, if we let $z = \frac{1}{x}$, $\lim_{x\to\infty} \frac{x+3}{x^2+7} = \lim_{z\to 0} \frac{\frac{1}{z}+3}{\frac{1}{z^2}+7} = \lim_{z\to 0} \frac{z+3z^2}{1+7z^2} = 0$

(A) 0

(B) $\frac{3}{7}$

(C) 1

(D) ∞

(E) The limit does not exist and is not ∞

13. *The function* $f(x) = \begin{cases} \frac{x+5}{(x-2)(x-4)} & \textit{if } x \leqslant 3 \\ \frac{1-x^2}{x-2} & \textit{if } x > 3 \end{cases}$ *is continuous:*

On $(-\infty, 3)$, $f(x)$ is clearly continuous everywhere except at $x = 2$. On $(3, \infty)$, $f(x)$ is continuous everywhere, so the only remaining question is whether $f(x)$ is continuous at $x = 3$. Now, $\lim_{x\to 3^+} f(x) = \frac{1-3^2}{3-2} = -8$, and $f(3) = \lim_{x\to 3^-} f(x) = \frac{3+5}{(3-2)(3-4)} = -8$, so f is continuous at 3.

(A) everywhere.

(B) everywhere except at $x = 3$.

(C) everywhere except at $x = -5$ and $x = 2$.

(D) everywhere except at $x = 2$, $x = 3$, $x = 4$.

(E) everywhere except at $\mathbf{x = 2}$.

14. *The derivative of* $\sqrt[3]{x^2 + 7x - 3}$ *at* $x = 3$ *is:*

Use the chain rule with $u = x^2 + 7x - 3$ and $y = \sqrt[3]{u}$. We have that when $x = 3$, $u = 27$, and $\frac{dy}{du} = \frac{1}{3}u^{-\frac{2}{3}} = \frac{1}{27}$, and $\frac{du}{dx} = 2x + 7 = 13$, so by the chain rule, $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = \frac{1}{27} \times 13 = \frac{13}{27}$.

(A) $-\frac{13}{27}$

(B) $\frac{13}{27}$

(C) $\frac{13}{9}$

(D) 39

(E) undefined

15. *A manufacturer is selling a product. The demand equation is given by $p = 1000 - 5q - q^2$. The marginal revenue is given by:*

    Revenue is the product $r = pq = (1000 - 5q - q^2)q = 1000q - 5q^2 - q^3$. Marginal revenue is the derivative $\frac{dr}{dq} = 1000 - 10q - 3q^2$.

    (A) $-5 - 2q$

    (B) $1000 - 5q - q^2$

    (C) $\mathbf{1000 - 10q - 3q^2}$

    (D) $\frac{-1000}{q^2} - 1$

    (E) 1000

16. *For the demand function in the previous question, the point elasticity of demand at $p = 500, q = 20$ is:*

    Point elasticity of demand is given by $\eta = \frac{\left(\frac{p}{q}\right)}{\left(\frac{dp}{dq}\right)}$. We have $\frac{dp}{dq} = -5 - 2q$, so when $q = 20$, $\eta = \frac{\left(\frac{500}{20}\right)}{-45} = -\frac{25}{45} = -\frac{5}{9}$.

    (A) $\frac{5}{9}$

    (B) $-\frac{5}{9}$

    (C) -25

    (D) $-\frac{1}{25}$

    (E) -1

17. *The function $f(x)$ satisfies $f'(5) = 0$ and $f''(5) = 2$. The point $x = 5$ is*

    (A) A local maximum for $f(x)$.

    (B) A local minimum for f(x).

    (C) Definitely a global maximum for $f(x)$.

    (D) Definitely a global minimum for $f(x)$.

    (E) Neither a local maximum nor a global minimum.

18. *The critical value of the function $f(x) = \frac{x^2-3}{x+2}$ at $x = -1$ is:*

    By the quotient rule,

    $f'(x) = \frac{(2x)(x+2)-(x^2-3)}{(x+2)^2} = \frac{x^2+4x+3}{(x+2)^2}$. (So we can check $f(-1) = 0$, so $x = -1$ is indeed a critical value.) $f''(x) = \frac{(2x+4)(x+2)^2-(x^2+4x+3)2(x+2)}{(x+2)^4}$, so $f''(-1) = \frac{2\times 1^2 - 0\times 2\times 1}{1^4} = 2 > 0$, so $x = -1$ is a local minimum.

$f(x)$ has a vertical asymptote at $x = -2$, with $\lim_{x \to -2^-} f(x) = -\infty$ and $\lim_{x \to -2^+} f(x) = \infty$, so f has no global maximum or minimum.

(A) a local (relative) maximum, but not a global (absolute) maximum.

(B) a global (absolute) maximum.

(C) a local (relative) minimum, but not a global (absolute) minimum.

(D) a global (absolute) minimum.

(E) Neither a local minimum nor a local maximum.

19. *Which of the following lines is an asymptote to the function* $f(x) = \frac{x^4-5x+3}{x^2-3}$ *?*

$f(x)$ has a vertical asymptote when the denominator is zero, i.e. when $x^2 = 3$.

(A) $x = \sqrt{3}$

(B) $y = 3$

(C) $y = 2x + 3$

(D) None of these, but $f(x)$ does have an asymptote.

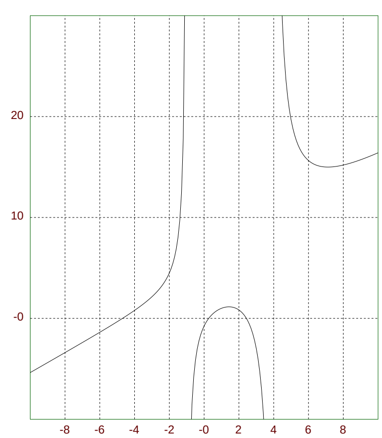
(E) $f(x)$ does not have any asymptotes.

20. *Let* $f(x) = \frac{2x^3+10x^2+5x-5}{(x+1)^2(x-4)}$. *It is possible to calculate* $f'(x) = -\frac{14x^3+24x^2+45x+55}{(x+1)^3(x-4)^2}$ *so* $f'(x) = 0$ *has a solution at -1.41... and* $f''(x) = -\frac{28x^4+58x^3+252x^2+152x-370}{(x+1)^4(x-4)^3}$ *so* $f''(x) = 0$ *has solutions at 0.86... and -1.6325...*

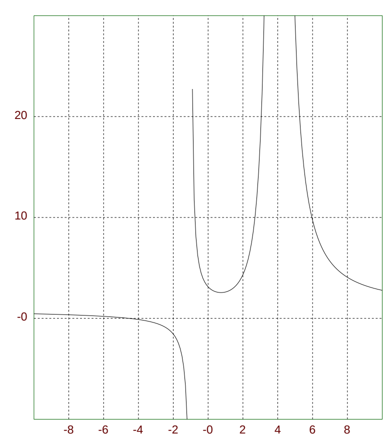
Which of the following is the graph of $f(x)$*?*

From the information given, we know that $f(x)$ has a local maximum or minimum at $x = -1.41...$. (It must be a maximum or minimum, rather than a point of inflection, because $f''(-1.41...) \neq 0$.) The only graph with this property is (E).

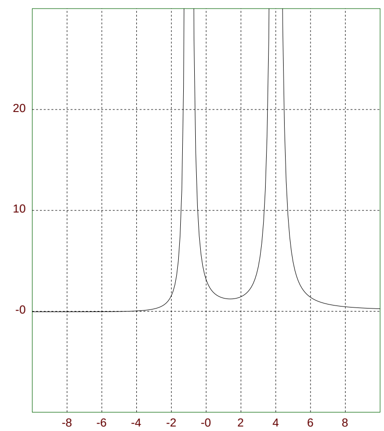
(A)



(B)

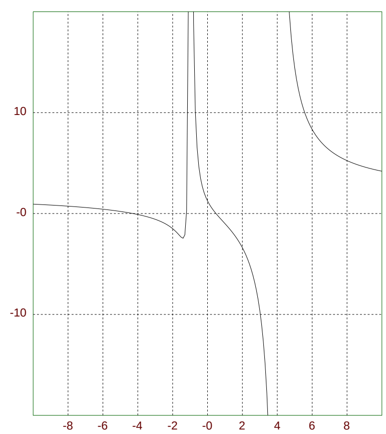


(C)



(D)

(E)



21. Solve the system of equations

$$\begin{array}{rrrcr} a & +2b & -3c & = & 4 \\ 3a & +5b & -7c & = & 12 \\ 2a & +5b & -5c & = & 2 \end{array}$$

We write out the augmented coefficient matrix

$$\left(\begin{array}{rrr|r} 1 & 2 & -3 & 4 \\ 3 & 5 & -7 & 12 \\ 2 & 5 & -5 & 2 \end{array}\right)$$

We subtract 3 times the first row from the second row and subtract 2 times the first row from the third row to get:

$$\left(\begin{array}{rrr|r} 1 & 2 & -3 & 4 \\ 0 & -1 & 2 & 0 \\ 0 & 1 & 1 & -6 \end{array}\right)$$

We divide the second row by -1, then subtract twice the second row from the first row, and subtract the second row from the third row.

$$\left(\begin{array}{rrr|r} 1 & 0 & 1 & 4 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 3 & -6 \end{array}\right)$$

Finally we divide the last row by 3, and add 2 times the last row to the second row and subtract the last row from the first row to get

$$\left(\begin{array}{rrr|r} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & -2 \end{array}\right)$$

The solution is therefore $x = 6$, $y = -4$, $z = -2$.

We check:

$$\begin{array}{rrrcr} 6 & +2 \times -4 & -3 \times -2 & = & 4 \\ 3 \times 6 & +5 \times -4 & -7 \times -2 & = & 12 \\ 2 \times 6 & +5 \times -4 & -5 \times -2 & = & 2 \end{array}$$

22. *Given the following table, find a set of simultaneous equations that need to be solved in order to calculate the number of units of each sector that must be produced in order to meet the external demand of 20 units of sector A, 30 units of sector B and 30 units of sector C.*

	Sector A	*Sector B*	*Sector C*	*External Demand*	*Total*
Sector A	*100*	*300*	*150*	*450*	*1000*
Sector B	*300*	*600*	*150*	*150*	*1200*
Sector C	*50*	*150*	*150*	*400*	*750*
Other costs	*700*	*150*	*300*		
Total	*1000*	*1200*	*750*		

We obtain the Leontieff matrix A by dividing each element in the table by the total for the corresponding column, to get

$$A = \begin{pmatrix} \frac{1}{10} & \frac{1}{4} & \frac{1}{5} \\ \frac{3}{10} & \frac{1}{2} & \frac{1}{5} \\ \frac{1}{20} & \frac{1}{8} & \frac{1}{5} \end{pmatrix}$$

We need to solve $(I-A)x = d$, where $d = \begin{pmatrix} 20 \\ 30 \\ 30 \end{pmatrix}$. We have that

$$I - A = \begin{pmatrix} \frac{9}{10} & -\frac{1}{4} & -\frac{1}{5} \\ -\frac{3}{10} & \frac{1}{2} & -\frac{1}{5} \\ -\frac{1}{20} & -\frac{1}{8} & \frac{4}{5} \end{pmatrix}$$

This gives us the simultaneous equations

$$\begin{array}{ccccc} \frac{9}{10}a & -\frac{1}{4}b & -\frac{1}{5}c & = & 20 \\ -\frac{3}{10}a & +\frac{1}{2}b & -\frac{1}{5}c & = & 30 \\ -\frac{1}{20}a & -\frac{1}{8}b & +\frac{4}{5}c & = & 30 \end{array}$$

23. *Find the maximum of $f(x) = x^3 - x^2 + 7$ on the interval $[-1, 1]$. Justify why your answer must be the maximum.*

$f'(x) = 3x^2 - 2x$ is defined everywhere on the interval $[-1, 1]$. The possible locations of the maximum for $f(x)$ are therefore:

- The solutions to $f'(x) = 0$
- The endpoints $x = -1$ and $x = 1$.

It is easy to see that the solutions to $f'(x) = 0$ are $x = 0$ and $x = \frac{2}{3}$. We therefore just need to check our 4 possible solutions to find the maximum value of $f(x)$.

x	$f(x)$
-1	5
0	7
$\frac{2}{3}$	$\frac{185}{27}$
1	7

Therefore, we see that $x = 0$, $f(x) = 7$ and $x = 1$, $f(x) = 7$ are the maxima.

An alternative way to rule out $\frac{2}{3}$ is using the second derivative test: $f''(x) = 6x - 2$, so $f''\left(\frac{2}{3}\right) = 2 > 0$, so $x = \frac{2}{3}$ is a local minimum, and therefore cannot be a global maximum.

24. *Find all asymptotes to the function $f(x) = \frac{x^3-5x+3}{x^2+2x+4}$.*

There are vertical asymptotes whenever the denominator is 0, but this never happens since $x^2 + 2x + 4 = (x+1)^2 + 3$. [Alternatively, by taking the derivative of x^2+2x+4, we see that it has a local minimum at $x=-1$, where its value is 3, and since it has no local maxima, this local minimum must be the global minimum.]

There are therefore no vertical asymptotes, and we need to look for non-vertical asymptotes. We can calculate $f(x) = x - 2 - \frac{5x-5}{x^2+2x+4}$. However the final fraction has a limit 0 when x tends to $\pm\infty$. Therefore, the line $y = x - 2$ is the only asymptote to the curve.

25. *For two products, A and B, the demand functions for the products are given by:*

$$q_A = 1000 - 3p_A - p_B \tag{1}$$
$$q_B = \frac{500}{p_A + 10p_B} \tag{2}$$

(a) Calculate the partial derivatives $\frac{\partial q_A}{\partial p_A}$, $\frac{\partial q_A}{\partial p_B}$, $\frac{\partial q_B}{\partial p_A}$, and $\frac{\partial q_B}{\partial p_B}$. Determine whether the products are competitive or complementary (or neither).

$$\frac{\partial q_A}{\partial p_A} = -3$$
$$\frac{\partial q_A}{\partial p_B} = -1$$
$$\frac{\partial q_B}{\partial p_A} = -\frac{500}{(p_A + 10p_B)^2}$$
$$\frac{\partial q_B}{\partial p_B} = -\frac{5000}{(p_A + 10p_B)^2}$$

Since $\frac{\partial q_A}{\partial p_B} < 0$ and $\frac{\partial q_B}{\partial p_B} < 0$, the products are complementary.

(b) Suppose the products are both produced by the same company. Now what equations should be solved to find the prices p_A and p_B that the company should charge to maximise its total revenue. [You do not need to solve these equations, but you should simplify them to equations which involve just p_A and p_B.]

To maximise revenue, $r = r_A + r_B$. We need to solve

$$\frac{\partial r}{\partial p_A} = 0$$
$$\frac{\partial r}{\partial p_B} = 0$$

We know $r_A = p_A q_A = p_A(1000 - 3p_A - p_B)$, so $\frac{\partial r_A}{\partial p_A} = 1000 - 6p_A - p_B$ and $\frac{\partial r_A}{\partial p_B} = p_A$. Also, $r_B = p_B q_B = \frac{500 p_B}{p_A + 10 p_B}$, so $\frac{\partial r_B}{\partial p_A} = -\frac{500 p_B}{(p_A + 10 p_B)^2}$ and $\frac{\partial r_B}{\partial p_B} = \frac{500(p_A + 10 p_B) - 500 p_B \times 10}{(p_A + 10 p_B)^2} = \frac{500 p_A}{(p_A + 10 p_B)^2}$.

Therefore, $\frac{\partial r}{\partial p_A} = 1000 - 6p^A - p_B - \frac{500 p_B}{(p_A + 10 p_B)^2}$ and $\frac{\partial r}{\partial p_B} = p_A + \frac{500 p_A}{(p_A + 10 p_B)^2}$. That is, we need to find the solution to the equations:

$$\begin{aligned} 1000 - 6p^A - p_B - \frac{500 p_B}{(p_A + 10 p_B)^2} &= 0 \\ p_A + \frac{500 p_A}{(p_A + 10 p_B)^2} &= 0 \end{aligned}$$