MATH 1115, Mathematics for Commerce
WINTER 2011
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Homework Sheet 3
Due: Wednesday 9th February: 2:30 PM

Each multiple choice question is worth one mark, other questions are worth two marks.

1. Which of the following satisfies the system of inequalities

$$\begin{aligned} x + 2y - 3z &\leqslant 5 \\ 2x - 2y + z &\leqslant 2 \\ x + 2y - 3z &\geqslant 1 \\ 4x - 2y - 2z &\geqslant -2 \end{aligned}$$

   (A) $x = 4, y = 6, z = 5$

   (B) $x = 1, y = 2, z = -1$

   (C) $x = 4, y = 0, z = 2$

   (D) $x = -2, y = 3, z = -1$

   (E) None of the above

2. The feasible region for the system of inequalities

$$\begin{aligned} 2x + 2y &\leqslant 8 \\ x - y &\leqslant 2 \\ x + 2y &\geqslant 1 \\ -2x + 4y &\geqslant 3 \end{aligned}$$

   is:

   (A) Bounded and $x = 3, y = 1$ is a corner.

   (B) Empty

   (C) Unbounded and $x = 7, y = -3$ is a corner.

   (D) Bounded and $x = 7, y = -3$ is a corner.

   (E) Unbounded and $x = 3, y = 1$ is a corner.

3. The maximum value of $5x + y$ subject to the constraints

$$\begin{aligned} 2x + y &\leqslant 6 \\ -x + y &\leqslant 2 \\ x - 2y &\geqslant 1 \\ x + 3y &\geqslant 3 \end{aligned}$$

is

(A) Attained at $x = 1.8, y = 0.4$

(B) 10

(C) Attained at $x = 2.6, y = 0.8$

(D) 15

(E) Undefined. (That is, there is no maximum value).

4. While applying the simplex method to solve a linear programming problem to maximise Z, you obtain the following simplex tableau.

x_1	x_2	x_3	s_1	s_2	s_3	s_4	Z	
1	0	-2	0	0	3	0	0	5
0	3	-3	1	0	2	0	0	3
0	2	4	0	1	-3	0	0	8
0	-1	2	0	0	0	1	0	11
0	-1	-2	0	0	3	0	1	15

Increasing which of the non-basic variables in this tableau would increase the value of Z?

(A) x_1, x_2 or x_3

(B) x_2, x_3 or s_3

(C) Just x_3

(D) x_2 or x_3

(E) None of them — the current point is the one which maximises Z.

5. While applying the simplex method to solve a linear programming problem to maximise Z, you obtain the following simplex tableau.

x_1	x_2	x_3	s_1	s_2	s_3	s_4	Z	
1	0	-2	0	0	4	0	0	5
0	4	1	1	0	1	0	0	3
0	2	-2	0	1	-1	0	0	8
0	-2	1	0	0	2	1	0	11
0	-2	3	0	0	-1	0	1	15

You decide that the entering variable should be x_2. The value of x_2 in the next BFS, and the departing variable are:

	New x_2 value	Departing variable
(A)	0.75	s_1
(B)	0.75	s_3
(C)	4	s_2
(D)	0	x_1
(E)	-5.5	s_4

6. While applying the simplex method to solve a linear programming problem to maximise Z, you obtain the following simplex tableau.

x_1	x_2	x_3	s_1	s_2	s_3	s_4	Z	
1	0	-2	0	0	4	0	0	5
0	4	1	1	0	1	0	0	3
0	2	-2	0	1	-1	0	0	8
0	-2	1	0	0	2	1	0	11
0	-2	3	0	0	-1	0	1	15

What is the current BFS?

(A) $x_1 = 0,\ x_2 = 0,\ x_3 = 0,\ s_1 = 5,\ s_2 = 3,\ s_3 = 8,\ s_4 = 11.$

(B) $x_1 = 5,\ x_2 = 0,\ x_3 = 0,\ s_1 = 3,\ s_2 = 8,\ s_3 = 0,\ s_4 = 11.$

(C) $x_1 = 3,\ x_2 = 0.75,\ x_3 = -4,\ s_1 = 0,\ s_2 = 0,\ s_3 = 0,\ s_4 = 11.$

(D) $x_1 = 0,\ x_2 = 5,\ x_3 = 3,\ s_1 = 0,\ s_2 = 0,\ s_3 = 8,\ s_4 = 0.$

(E) The current BFS cannot be determined from this tableau.

7. Use the simplex method to solve the following problem:

Maximise $Z = x_1 + x_2 + x_3$

Subject to:

$$
\begin{aligned}
3x_1 + x_2 &\leqslant 6 \\
-x_1 + x_2 &\leqslant 2 \\
x_1, x_2 &\geqslant 0
\end{aligned}
$$

8. A company manufactures two different types of product, A and B. Each product requires the following resouces:

Product	A	B
Raw material X	5	3
Employee hours	1	3
Machine hours	2	5
Profit	15	25

The company has a total of 300 units of raw material available, a total of 100 employee hours, and a total of 250 machine hours available for production of these products. Furthermore, the company estimates the total demand for product A to be 200, and for product B to be 80. The manager wants to determine how many of each type of product should be produced in order to maximise profit.

Write out the linear programming problem that she should solve in order to calculate the maximum profit. [You do not need to actually calculate the maximum profit, just write down the problem that needs to be solved.]

9. Convert the following linear programming problem into standard form

   Minimise $Z = x_1 - 2x_2 + x_3$, subject to:

$$\begin{aligned} 2x_1 + x_2 - x_3 &\leqslant 6 \\ -x_1 + 3x_2 - 2x_3 &\leqslant 2 \\ 2x_2 - 4x_3 &\geqslant -3 \\ x_1 - 3x_2 &\geqslant -1 \\ x_1, x_2, x_3 &\geqslant 0 \end{aligned}$$

   [You do not need to solve the problem after converting it to standard form.]