MATH 1115, Mathematics for Commerce
WINTER 2011
Toby Kenney
Homework Sheet 9
Due: Wednesday 6th April: 2:30 PM

Each multiple choice question is worth one mark, other questions are worth two marks. Show your working for the other questions, but for multiple choice questions, just the letter is sufficient.

1. The second derivative of $2x^4 - 3x^2 + 7x + 2$ is:
   (A) $8x^3 - 6x + 7$
   (B) $(8x^3 - 6x + 7)^2$
   (C) $6x^2 - 3$
   (D) $24x^2 - 6$
   (E) undefined

2. The function $f(x) = x^4 + 2x^3 - 7x + 4$ is:
   (A) increasing at both $x = -1$ and at $x = 1$.
   (B) increasing at $x = -1$ but decreasing at $x = 1$.
   (C) decreasing at $x = -1$ but increasing at $x = 1$.
   (D) decreasing at both $x = -1$ and at $x = 1$.
   (E) at a local extremum at one of the points $x = 1$ and $x = -1$.

3. The critical value of the function $f(x) = \frac{x^2-2x+1}{x+3}$ at $x = 1$ is:
   (A) a local (relative) maximum, but not a global (absolute) maximum.
   (B) a global (absolute) maximum.
   (C) a local (relative) minimum, but not a global (absolute) minimum.
   (D) a global (absolute) minimum.
   (E) Neither a local minimum nor a local maximum.

4. Which of the following lines is an asymptote to the function $f(x) = \frac{x^4-2x+3}{x^2+3}$?
   (A) $x = \sqrt{3}$
   (B) $y = 3$
   (C) $y = 2x + 3$
   (D) None of these, but $f(x)$ does have an asymptote.
   (E) $f(x)$ does not have any asymptotes.

5. A company's revenue, r as a function of the amount of production x is given by $r = 2x + 10\sqrt{x}$. Meanwhile the cost of production, c is given by $c = \frac{3x^2+13x}{x+1}$. The amount of production x which maximises the company's profit is a solution to (Assuming there is some x which maximises profit):

   (A) $2x + 10\sqrt{x} = \frac{3x^2+13x}{x+1}$

   (B) $2 + \frac{5}{\sqrt{x}} = 0$

   (C) $2 + \frac{5}{\sqrt{x}} = \frac{3x^2+6x+13}{(x+1)^2}$

   (D) $(2x + 10\sqrt{x})\left(\frac{3x^2+6x+13}{(x+1)^2}\right) = \left(2 + \frac{5}{\sqrt{x}}\right)\left(\frac{3x^2+13x}{x+1}\right)$

   (E) $-\frac{5}{\sqrt{(x)}} = -\frac{10}{(x+1)^2}$

   [Bonus question: What do the the solutions to the wrong answers in Q. 5 (assuming they exist) correspond to?]

6. If $z = x^2 + 3xy + 2xy^2 - 3y^3$, then $\frac{\partial z}{\partial y}$ at $x = 1$, $y = 2$ is

   (A) -25

   (B) -24

   (C) -9

   (D) 1

   (E) 16

7. For two products, A and B, the demand functions for the products are given by:

$$q_A = 1000 - \frac{p_A}{2000 - p_B} \tag{1}$$

$$q_B = \frac{300}{p_A + p_B} \tag{2}$$

   (a) Calculate the partial derivatives $\frac{\partial q_A}{\partial p_A}$, $\frac{\partial q_A}{\partial p_B}$, $\frac{\partial q_B}{\partial p_A}$, and $\frac{\partial q_B}{\partial p_B}$. Determine whether the products are competitive or complementary (or neither).

   (b) Suppose the products are produced by two different companies. Each company sets the price for their own product. What equations need to be solved so that neither company can increase their own revenue by changing price? [You do not need to solve the equations, but you must simplify them to equations involving only p_A and p_B. Hint: There are two equations that must be satisfied.]

   (c) Now suppose the products are both produced by the same company. Now what equations should be solved to find the prices p_A and p_B that the company should charge to maximise its total revenue. [Again, the equations should involve just p_A and p_B.]