MATH 1115, Mathematics for Commerce
WINTER 2011
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Homework Sheet 1
Model Solutions

Each multiple choice question is worth one mark, other questions are worth two marks.

1. Which of the following rates of interest is best for the investor, in the long term?

   (A) 6.85% compounded daily

   =7.09% annual effective rate.

   (B) 7% compounded monthly

   =7.23% annual effective rate.

   (C) 7.1% compounded quarterly

   =7.29% annual effective rate.

   (D) 7.2% compounded semi-annually

   =7.33% annual effective rate.

   (E) 7.25% compounded annually

2. In a 20-year mortgage for \$200,000 with interest 6% compounded monthly, the monthly repayments are given by:

   (A) $\frac{200000\times 1.06}{20}$

   (B) $\frac{200000\times 1.06}{240}$

   (C) $200000\frac{1-1.005^{-240}}{0.005}$

   (D) $\boldsymbol{\frac{200000\times 0.005}{1-1.005^{-240}}}$

   (E) $200000\frac{1.005^{240}-1}{0.005}$

3. If you invest \$200 at the end of every month for 5 years, at interest rate 9% compounded monthly, the accumulated value of the investments after the last payment is made is:

   (A) $\boldsymbol{200\frac{1.0075^{60}-1}{0.0075}}$

   (B) $200\frac{1-1.0075^{-60}}{0.0075}$

   (C) $200 \times 60 \times 1.09^{2.5}$

   (D) $200 \times 60 \times 1.0075^{30}$

   (E) $(200 \times 1.0075^{60} + 200) \times 30$

4. The number of years that it takes for money to double in value when invested at 6% compounded monthly is given by:

   (A) $\frac{2}{1.06}$

   (B) $\frac{1}{0.06}$

   (C) $\frac{\ln 2}{\ln 0.06}$

   (D) $\boldsymbol{\frac{\ln 2}{12\times\ln 1.005}}$

   (E) $\frac{\ln 2}{\ln 1.06}$

5. The number of monthly repayments of \$500 that are needed to pay off a loan for \$20,000 at interest rate 7% compounded monthly is given by:

   (A) $\frac{20000\times 1.07}{500}$

   (B) $\frac{20000\times(1+\frac{0.07}{12})^{12}}{500}$

   (C) $\frac{\ln(40)}{\ln\left(1+\frac{0.07}{12}\right)}$

   (D) $\frac{\ln\left(1-\frac{40\times 0.07}{12}\right)}{\ln\left(1+\frac{0.07}{12}\right)}$

   (E) $\boldsymbol{-\frac{\ln\left(1-\frac{40\times 0.07}{12}\right)}{\ln\left(1+\frac{0.07}{12}\right)}}$

6. A loan for \$500 at 12% interest compounded monthly, is repaid by monthly payments starting one year after the loan is taken out, and continuing for the following 12 months (so there are 13 payments in total). The monthly repayments R are given by solving the equation:

   (A) $500 = R(1.01)^{-12}\frac{1-1.01^{-13}}{0.01}$

   (B) $\boldsymbol{500 = \mathbf{R}(1.01)^{-11}\frac{1-1.01^{-13}}{0.01}}$

   (C) $500 = R\frac{1-1.01^{-13}}{0.01}$

   (D) $500 = R(1.01)^{11}\frac{1-1.01^{-13}}{0.01}$

   (E) $500 = R(1.01)^{12}\frac{1-1.01^{-13}}{0.01}$

7. Suppose you want to invest \$200 a month into your pension fund, which receives interest at 3% compounded monthly, starting one month after your 30th birthday and ending when you retire, and suppose you want to be able to withdraw \$1500 a month from this fund after you retire, until you reach the age of 90. The equation for the number of months n from your 30th birthday until you can retire is:

   (A) $200\frac{1-1.0025^{-n}}{0.0025} = 1500\frac{1-1.0025^{(60\times 12)-n}}{0.0025}$

   (B) $\boldsymbol{200\frac{1.0025^{n}-1}{0.0025} = 1500\frac{1-1.0025^{-((60\times 12)-n)}}{0.0025}}$

   (C) $200\frac{1.0025^{n}-1}{0.0025} = 1500\frac{1-1.0025^{-n}}{0.0025}$

   (D) $200\frac{1.0025^{n}-1}{0.0025} = 1500\frac{1.0025^{(60\times 12)-n}-1}{0.0025}$

(E) $200\frac{1-1.0025^{-n}}{0.0025} = 1500\frac{1-1.0025^{-n}}{0.0025}$

8. *You wish to set up a scholarship fund which will pay out \$20,000 a year. If the money can be invested at 5% compounded annually, how much do you need to invest in this fund one year before the first payment is to be made?*

   This is a perpetuity, which at 5% interest compounded annually, pays out \$20,000 a year. The formula is $R = Pi$, where $R = 20000$ is the regular payment, P is the present value one period before the first payment, and $i = 5\%$ is the interest rate per period. This gives $20000 = 0.05P$, or $P = \$400,000$.

9. *For a \$300,000 mortgage with interest 12% compounded monthly, to be repaid with monthly repayments over 20 years, the monthly repayments are \$3303.26, and the final repayment is \$3301.68. Calculate the outstanding balance after 5 years, and the amount of interest included in the following payment (i.e. the payment in 5 years and 1 month's time).*

   Solution 1:

   Looking at the original loan, and the payments already made, the original \$300,000 has value $300000(1.01)^{60}$ after 5 years. The payments made during these 5 years have value $3303.26\frac{1.01^{60}-1}{0.01}$, so the outstanding balance is given by $300000(1.01)^{60} - 3303.26\frac{1.01^{60}-1}{0.01} = \$275,232.86$.

   Solution 2:

   Looking at the remaining payments, these form an annuity with regular payments \$3303.26 for 15 years, with a discount of \$1.58 on the final payment. The present value of the annuity is $3303.26\frac{1-1.01^{-180}}{0.01}$, and the present value of the discount is 1.58×1.01^{-180} so the outstanding balance is given by $3303.26\frac{1-1.01^{-180}}{0.01} - 1.58 \times 1.01^{-180} = \$275,232.86$.

   The interest portion of the following monthly payment is 1% of the outstanding balance, so it is \$2,752.33.