MATH 1115, Mathematics for Commerce
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Homework Sheet 3
Model Solutions

Each multiple choice question is worth one mark, other questions are worth two marks.

1. Which of the following satisfies the system of inequalities

$$\begin{aligned} x + 2y - 3z &\leqslant 5 \\ 2x - 2y + z &\leqslant 2 \\ x + 2y - 3z &\geqslant 1 \\ 4x - 2y - 2z &\geqslant -2 \end{aligned}$$

(A) $x = 4, y = 6, z = 5$

(B) $x = 1, y = 2, z = -1$

(C) $x = 4, y = 0, z = 2$

(D) $x = -2, y = 3, z = -1$

(E) None of the above

2. The feasible region for the system of inequalities

$$\begin{aligned} 2x + 2y &\leqslant 8 \\ x - y &\leqslant 2 \\ x + 2y &\geqslant 1 \\ -2x + 4y &\geqslant 3 \end{aligned}$$

is:

(A) Bounded and $x = 3, y = 1$ is a corner.

(B) Empty

(C) Unbounded and $x = 7, y = -3$ is a corner.

(D) Bounded and $x = 7, y = -3$ is a corner.

(E) Unbounded and $x = 3, y = 1$ is a corner.

None of these answers is correct, due to a typo in the question. I had intended the second inequality to read $x - y \geqslant 2$, which would have made (B) the answer. As it stands, the feasible region is unbounded, but neither $x = 3, y = 1$ nor $x = 7, y = -1$ are corners.

3. The maximum value of $5x + y$ subject to the constraints

$$2x + y \leqslant 6$$
$$-x + y \leqslant 2$$
$$x - 2y \geqslant 1$$
$$x + 3y \geqslant 3$$

is

(A) Attained at $x = 1.8, y = 0.4$

(B) 10

(C) Attained at $x = 2.6, y = 0.8$

(D) 15

(E) Undefined. (That is, there is no maximum value).

4. While applying the simplex method to solve a linear programming problem to maximise Z, you obtain the following simplex tableau.

x_1	x_2	x_3	s_1	s_2	s_3	s_4	Z	
1	0	-2	0	0	3	0	0	5
0	3	-3	1	0	2	0	0	3
0	2	4	0	1	-3	0	0	8
0	-1	2	0	0	0	1	0	11
0	-1	-2	0	0	3	0	1	15

Increasing which of the non-basic variables in this tableau would increase the value of Z?

(A) x_1, x_2 or x_3

(B) x_2, x_3 or s_3

(C) Just x_3

(D) $\mathbf{x_2}$ or $\mathbf{x_3}$

(E) None of them — the current point is the one which maximises Z.

5. While applying the simplex method to solve a linear programming problem to maximise Z, you obtain the following simplex tableau.

x_1	x_2	x_3	s_1	s_2	s_3	s_4	Z	
1	0	-2	0	0	4	0	0	5
0	4	1	1	0	1	0	0	3
0	2	-2	0	1	-1	0	0	8
0	-2	1	0	0	2	1	0	11
0	-2	3	0	0	-1	0	1	15

You decide that the entering variable should be x_2. The value of x_2 in the next BFS, and the departing variable are:

	New x_2 value	Departing variable
(A)	**0.75**	$\mathbf{s_1}$
(B)	0.75	s_3
(C)	4	s_2
(D)	0	x_1
(E)	-5.5	s_4

6. While applying the simplex method to solve a linear programming problem to maximise Z, you obtain the following simplex tableau.

x_1	x_2	x_3	s_1	s_2	s_3	s_4	Z	
1	0	-2	0	0	4	0	0	5
0	4	1	1	0	1	0	0	3
0	2	-2	0	1	-1	0	0	8
0	-2	1	0	0	2	1	0	11
0	-2	3	0	0	-1	0	1	15

What is the current BFS?

(A) $x_1 = 0,\ x_2 = 0,\ x_3 = 0,\ s_1 = 5,\ s_2 = 3,\ s_3 = 8,\ s_4 = 11.$

(B) $\mathbf{x_1 = 5,\ x_2 = 0,\ x_3 = 0,\ s_1 = 3,\ s_2 = 8,\ s_3 = 0,\ s_4 = 11.}$

(C) $x_1 = 3,\ x_2 = 0.75,\ x_3 = -4,\ s_1 = 0,\ s_2 = 0,\ s_3 = 0,\ s_4 = 11.$

(D) $x_1 = 0,\ x_2 = 5,\ x_3 = 3,\ s_1 = 0,\ s_2 = 0,\ s_3 = 8,\ s_4 = 0.$

(E) The current BFS cannot be determined from this tableau.

7. *Use the simplex method to solve the following problem:*

Maximise $Z = x_1 + x_2$

Subject to:

$$3x_1 + x_2 \leqslant 6$$
$$-x_1 + x_2 \leqslant 2$$
$$x_1, x_2 \geqslant 0$$

We start at the BFS $x_1 = 0$, $x_2 = 0$. Here, the simplex tableau is:

x_1	x_2	s_1	s_2	Z	
3	1	1	0	0	6
-1	1	0	1	0	2
-1	-1	0	0	1	0

We can choose either x_1 or x_2 as the entering model:

If we choose x_1:

we see that the next BFS is $x_1 = 2$, $x_2 = 0$, since only the first row has a positive coefficient of x_1, and the departing variable is s_1. We now rearrange the simplex tableau by dividing row 1 by 3, and adding the new row 1 to row 2 and row 3 to get:

x_1	x_2	s_1	s_2	Z	
1	$\frac{1}{3}$	$\frac{1}{3}$	0	0	2
0	$\frac{4}{3}$	$\frac{1}{3}$	1	0	4
0	$-\frac{2}{3}$	$\frac{1}{3}$	0	1	2

We deduce that x_2 must be the next entering variable, and the quotients for row 1 and row 2 are 6 and 3 respectively, so the next BFS must be $x_2 = 3$ (and therefore, $x_1 = 2 - 3 \times \frac{1}{3} = 1$).

If we choose x_2:

For the first row, the quotient is $\frac{6}{1} = 6$, while for the second row it is $\frac{2}{1} = 2$. We therefore see that the next BFS is $x_1 = 0$, $x_2 = 2$, and the departing variable is s_2. We now rearrange the simplex tableau by swapping the rows (this is not necessary, but it keeps the basic variables in order, which may help us to see what is going on) and then subtracting row 1 from row 2 and adding it to row 3 to get:

x_1	x_2	s_1	s_2	Z	
-1	1	0	1	0	2
4	0	1	-1	0	4
-2	0	0	1	1	2

We deduce that the next enterring variable must be x_1, and that the next BFS is at $x_1 = 1$, $x_2 = 3$

Whether we enterred x_1 or x_2 first, when we reach the BFS $x_1 = 1$, $x_2 = 3$, the simplex tableau is

x_1	x_2	s_1	s_2	Z	
1	0	$\frac{1}{4}$	$-\frac{1}{4}$	0	1
0	1	$\frac{1}{4}$	$\frac{3}{4}$	0	3
0	0	$\frac{1}{2}$	$\frac{1}{2}$	1	4

So this is the optimal solution, and the maximum value of Z is 4.

8. *A company manufactures two different types of product, A and B. Each product requires the following resouces:*

Product	*A*	*B*
Raw material X	*5*	*3*
Employee hours	*1*	*3*
Machine hours	*2*	*5*
Profit	*15*	*25*

The company has a total of 300 units of raw material available, a total of 100 employee hours, and a total of 250 machine hours available for production of these products. Furthermore, the company estimates the total demand for product A to be 200, and for product B to be 80. The manager wants to determine how many of each type of product should be produced in order to maximise profit.

Write out the linear programming problem that she should solve in order to calculate the maximum profit. [You do not need to actually calculate the maximum profit, just write down the problem that needs to be solved.]

Let a be the number of items of product A produced, and let b be the number of items of product B produced. The problem to be solved is:

Maximise $15a + 25b$.

Subject to:

$$\begin{aligned} a &\leqslant 200 \\ b &\leqslant 80 \\ 5a + 3b &\leqslant 300 \\ a + 3b &\leqslant 100 \\ 2a + 5b &\leqslant 250 \\ a &\geqslant 0 \\ b &\geqslant 0 \end{aligned}$$

[The first inequality is total demand for A, the second is total demand for B, the third is raw material, the fourth is employee hours, the fifth is machine hours, and the sixth and seventh represent the fact that it is not possible to manufacture a negative number of something.]

9. *Convert the following linear programming problem into standard form*

   Minimise $Z = x_1 - 2x_2 + x_3$, *subject to:*

$$
\begin{aligned}
2x_1 + x_2 - x_3 &\leqslant 6 \\
-x_1 + 3x_2 - 2x_3 &\leqslant 2 \\
2x_2 - 4x_3 &\geqslant -3 \\
x_1 - 3x_2 &\geqslant -1 \\
x_1, x_2, x_3 &\geqslant 0
\end{aligned}
$$

[You do not need to solve the problem after converting it to standard form.]

Maximise $P = -x_1 + 2x_2 - x_3$, subject to:

$$
\begin{aligned}
2x_1 + x_2 - x_3 &\leqslant 6 \\
-x_1 + 3x_2 - 2x_3 &\leqslant 2 \\
-2x_2 + 4x_3 &\leqslant 3 \\
-x_1 + 3x_2 &\leqslant 1 \\
x_1, x_2, x_3 &\geqslant 0
\end{aligned}
$$