MATH 1115, Mathematics for Commerce
WINTER 2011
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Homework Sheet 4
Model Solutions

Each multiple choice question is worth one mark, other questions are worth two marks.

1. A business makes 3 kinds of product. These products require 4 different kinds of components. The number of each type of component required to make each product is represented by the table

	Component A	Component B	Component C	Component D
Product 1	1	2	0	3
Product 2	3	0	4	0
Product 3	2	2	1	1

These 4 components are made from 3 different kinds of raw materials. The matrix that gives the quantity of each raw material needed for each component is given by the table

	Raw material X	Raw material Y	Raw material Z
Component A	0	2	3
Component B	1	5	1
Component C	2	2	0
Component D	1	1	1

The cost per unit for each raw material is given by the table

Raw material X	20
Raw material Y	50
Raw material Z	5

The cost for raw materials for producing products 1, 2, and 3 are respectively:

(A) 525, 1085, and 655

(B) 720, 635 and 990

(C) 930, 855, and 890

(D) 890, 905, and 995

(E) 795, 910, and 840

2. For the system of equations:

$$\begin{array}{rrrcr} x & +3y & -z & = & 4 \\ 2x & -y & +z & = & 3 \\ 5x & +y & +z & = & 8 \end{array}$$

(A) The solution includes $x = 3$

(B) The solution includes $y = 4$

(C) The solution includes $z = 7$

(D) There is no solution.

(E) There are infinitely many solutions.

3. An economy with 3 sectors has Leontief matrix

$$A = \begin{pmatrix} 0.3 & 0.3 & 0.4 \\ 0.3 & 0.5 & 0.3 \\ 0.4 & 0.4 & 0.2 \end{pmatrix}$$

The production required to meet external demand $\begin{pmatrix} 30 \\ 20 \\ 40 \end{pmatrix}$ is:

(A) (-700 -800 -700)

(B) (700 800 700)

(C) (300 200 400)

(D) (45 -5 0)

(E) It is not possible to satisfy this external demand

4. The first row of the inverse of the matrix

$$A = \begin{pmatrix} 2 & 3 & 4 \\ 1 & 2 & 3 \\ 1 & 1 & 0 \end{pmatrix}$$

is:

(A) (3 -4 -1)

(B) (3 -3 1)

(C) $(\frac{1}{2}\ \frac{1}{3}\ \frac{1}{4})$

(D) (1 1 0)

(E)The matrix is not invertible

5. The maximum value of $2x + 4y$ subject to the constraints:

$$\begin{array}{rrcr} x & +2y & \leqslant & 4 \\ 2x & -y & \geqslant & 1 \\ 5x & +y & \leqslant & 15 \\ x, y \geqslant 0 & & & \end{array}$$

is:

(A) 8 and there is only one value of x, y where it is attained

(B) 6 and there is only one value of x, y where it is attained

(C) 8 and it is attained by infinitely many values of $\mathbf{x}, \mathbf{y}$.

(D) 6 and it is attained by infinitely many values of x, y.

(E) There is no maximum value

6. *(a) Write out an initial simplex tableau for the problem*

maximise $x + 2y + 4z$

subject to

$$\begin{array}{rrrcr} x & +3y & +z & \leqslant & 7 \\ 2x & -y & +3z & \leqslant & 8 \\ 5x & +y & -z & \leqslant & 15 \\ x & +y & +5z & \leqslant & 10 \\ x, y, z \geqslant 0 & & & & \end{array}$$

starting at the BFS $x = y = z = 0$.

Let $P = x + 2y + 4z$

x	y	z	s_1	s_2	s_3	s_4	P	
1	3	1	1	0	0	0	0	7
2	-1	3	0	1	0	0	0	8
5	1	-1	0	0	1	0	0	15
1	1	5	0	0	0	1	0	10
-1	-2	-4	0	0	0	0	1	0

(b) Use the simplex method to find the maximum value and the values of x, y *and* z *where it is attained.*

From the initial tableau, we choose z as the entering model [we could choose x or y instead, but we choose z because its entry in the bottom row is most negative.]

From the four rows, the maximum amounts by which we can increase z are 7, $\frac{8}{3}$, unlimited and 2 respectively. Therefore, the most we can increase x by is 2, and the departing variable is s_4. We now use row operations to get a new simplex tableau. We divide row 4 by 5, subtract the new row 4 from row 1, subtract 3 times the new row 4 from row 2, add the new row 4 to row 3, and add 4 times the new row 4 to row 5 to get:

x	y	z	s_1	s_2	s_3	s_4	P	
0.8	2.8	0	1	0	0	-0.2	0	5
1.4	-1.6	0	0	1	0	-0.6	0	2
5.2	1.2	0	0	0	1	0.2	0	17
0.2	0.2	1	0	0	0	0.2	0	2
-0.2	-1.2	0	0	0	0	0.8	1	8

From this, we decide that the next entering variable should be y. [x is also possible, but the entry for y is more negative, so we choose y.]

From the four rows, the maximum amounts by which we can increase y are $\frac{5}{2.8}$, unlimited, $\frac{17}{1.2}$ and 10 respectively. Therefore, the most we can increase y by is $\frac{5}{2.8}$, and the departing variable is s_1.

We now use row operations to get a new simplex tableau. We divide row 1 by 2.8, add 1.6 times the new row 1 to row 2, subtract 1.2 times the new row 1 from row 3, subtract 0.2 times the new row 1 to row 4, and add 1.2 times the new row 1 to row 5 to get:

x	y	z	s_1	s_2	s_3	s_4	P	
$\frac{2}{7}$	1	0	$\frac{1}{2.8}$	0	0	$-\frac{0.2}{2.8}$	0	$\frac{5}{2.8}$
$1.4+\frac{3.2}{7}$	0	0	$\frac{1.6}{2.8}$	1	0	$-0.6-\frac{0.32}{2.8}$	0	$2+\frac{8}{2.8}$
$5.2-\frac{1.2}{2.8}$	0	0	$-\frac{1.2}{2.8}$	0	1	$\frac{0.24}{2.8}+0.2$	0	$17-\frac{6}{2.8}$
$\frac{1}{7}$	0	1	$-\frac{0.2}{2.8}$	0	0	$0.2-\frac{0.04}{2.8}$	0	$2-\frac{1}{2.8}$
$\frac{2.4}{7}-0.2$	0	0	$\frac{1.2}{2.8}$	0	0	$0.8-\frac{0.24}{2.8}$	1	$8+\frac{6}{2.8}$

We can check that the entries on the bottom row are all non-negative, ($\geqslant 0$) so this BFS is the optimal solution.

The solution is:

$x=0$, $y=\frac{5}{2.8}=\frac{25}{14}$, $z=2-\frac{1}{2.8}=\frac{4.6}{2.8}=\frac{23}{14}$ and $P=8+\frac{6}{2.8}=\frac{28.4}{2.8}=\frac{71}{7}$.
[or to two decimal places, $x=0$, $y=1.79$, $z=1.64$ and $P=10.14$.]