MATH 1115, Mathematics for Commerce
WINTER 2011
Toby Kenney
Homework Sheet 8
Model Solutions

Each multiple choice question is worth one mark, other questions are worth two marks. Show your working for the other questions, but for multiple choice questions, just the letter is sufficient.

1. *The derivative of* $3x^3 + 4x^2 - 6x + 2$ *is:*

   (A) $3x^2 + 4x - 6$

   (B) $6x^2 + 4x - 6$

   (C) $\mathbf{9x^2 + 8x - 6}$

   (D) $9x^3 + 8x^2 - 6x + 2$

   (E) undefined

2. *The derivative of* $\frac{3x^2+2x-1}{x^2-5}$ *is:*

   Using the quotient rule with $f(x) = 3x^2 + 2x - 1$ and $g(x) = x^2 - 5$. The derivative of $\frac{f(x)}{g(x)}$ is

$$\frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2} = \frac{(6x+2)(x^2-5) - (3x^2+2x-1)(2x)}{(x^2-5)^2} =$$

$$\frac{6x^3 + 2x^2 - 30x - 10 - (6x^3 + 4x^2 - 2x)}{(x^2-5)^2} = \frac{-2x^2 - 28x - 10}{(x^2-5)^2}$$

   (A) $\frac{6x+2}{x^2-5}$

   (B) $\frac{6x+2}{2x}$

   (C) $-\frac{\mathbf{2x^2+28x+10}}{\mathbf{(x^2-5)^2}}$

   (D) $\frac{3x^3-6x^2+5x-3}{(x^2-5)^2}$

   (E) $-\frac{6x^3+2x^2-30x-10}{(x^2-5)^2}$

3. *The derivative of* $\sqrt{x^3 + 3x - 5}$ *at* $x = 2$ *is:*

   We use the chain rule. Let $u = x^3 + 3x - 5$ and let $y = \sqrt{u} = u^{\frac{1}{2}}$. Then $\frac{dy}{du} = \frac{1}{2}u^{-\frac{1}{2}}$ and $\frac{du}{dx} = 3x^2 + 3$. At $x = 2$, $\frac{du}{dx} = 3 \times 4 + 3 = 15$, while $u = 8 + 3 \times 2 - 5 = 9$, so $\frac{dy}{du} = \frac{1}{2\sqrt{u}} = \frac{1}{6}$, so by the chain rule, $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = \frac{15}{6} = \frac{5}{2}$.

Alternatively, we can calculate $\frac{dy}{dx}$ as a function of x using the chain rule.

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = \left(\frac{1}{2\sqrt{u}}\right)(3x^2+3) = \frac{3x^2+3}{2\sqrt{x^3+3x-5}} = \frac{5}{2}$$

(A) $-\frac{5}{2}$

(B) 3

(C) $\mathbf{\frac{5}{2}}$

(D) 45

(E) undefined

4. *A manufacturer is selling a product. The demand equation is given by* $p = \frac{1000}{q+3}$. *The marginal revenue is given by:*

   Revenue is given as the product pq. From the demand equation, this is equal to $\frac{1000}{q+3}q = \frac{1000q}{q+3}$. The marginal revenue is just the derivative of this, which, by the quotient rule is just $\frac{1000(q+3)-1000q\times 1}{(q+3)^2} = \frac{3000}{(q+3)^2}$.

   Alternatively, $\frac{1000q}{q+3} = \frac{1000(q+3)-3000}{q+3} = 1000 - \frac{3000}{q+3}$. Now we can take the derivative using the chain rule with $u = q+3$ and $y = 1000 - 3000u^{-1}$.

   (A) $-\frac{1000}{(q+3)^2}$

   (B) $\mathbf{\frac{3000}{(q+3)^2}}$

   (C) $\frac{1000}{q+3}$

   (D) $\frac{1000q}{q+3}$

   (E) $1000(q+3)$

5. *For the demand function in the previous question, the point elasticity of demand at* $p = 50, q = 17$ *is:*

   The elasticity of demand is given by $\frac{\left(\frac{p}{q}\right)}{\left(\frac{dp}{dq}\right)}$. For the demand function $p = \frac{1000}{q+3}$, we have that $\frac{dp}{dq} = -\frac{1000}{(q+3)^2}$ (using the chain rule with $u = q+3$). When $q = 17$, this gives $-\frac{1000}{20^2} = -\frac{5}{2}$. We therefore get

$$\eta = \frac{\left(\frac{50}{17}\right)}{\left(-\frac{5}{2}\right)} = -\frac{50\times 2}{5\times 17} = -\frac{20}{17}$$

   (A) $\frac{5}{2}$

   (B) $-\frac{2}{5}$

   (C) $\mathbf{-\frac{20}{17}}$

   (D) $-\frac{50}{17}$

   (E) $-\frac{17}{125}$

6. *A company estimates that the demand equation for its product is given by* $p = 10000 - q^2$ *for* $0 < q \leqslant 1000$.

(a) Calculate the elasticity of demand as a function of q.

The elasticity of demand is given by $\frac{\left(\frac{p}{q}\right)}{\left(\frac{dp}{dq}\right)}$. For the demand function $p = 10000 - q^2$, we have that $\frac{dp}{dq} = -2q$. Elasticity of demand is therefore given by

$$\frac{\left(\frac{p}{q}\right)}{\left(\frac{dp}{dq}\right)} = \frac{\left(\frac{10000-q^2}{q}\right)}{-2q} = -\frac{10000 - q^2}{2q^2}$$

(b) Calculate the price that the company should charge in order to maximise its revenue.

Revenue is maximised when $\eta = -1$, so in this case, when $-\frac{10000-q^2}{2q^2} = -1$, or $10000 - q^2 = 2q^2$, so $3q^2 = 10000$ or $q = \sqrt{\frac{10000}{3}}$. Using the demand equation, we see that the price that achieves this is $p = 10000 - q^2 = 10000 - \frac{10000}{3} = \frac{20000}{3}$.