MATH 2051, Problems in Geometry
Fall 2007
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Homework Sheet 7
Due in: Wednesday 14th November, 11:30 AM

Note that diagrams are not drawn to scale. Scale drawing does **not** constitute a proof. Justify all your answers.

Compulsory questions

1 (a) Show that the hyperbolic distance from the origin $(0,0)$ to the point $(0,x)$ is $2\tanh^{-1}x$. [Recall that the derivative of $\tanh x$ is $\frac{1}{\cosh^2 x}$, and that $1-\tanh^2 x = \frac{1}{\cosh^2 x}$.]

(b) Show that the (hyperbolic) circumference of a circle of hyperbolic radius r is $2\pi\sinh r$. [You may use the formula $\sinh 2x = 2\sinh x\cosh x$. Recall also that the circumference of a Euclidean circle of radius r is $2\pi r$.]

2 Find an explicit formula for an hyperbolic isometry sending the point $(0,0.5)$ to $(0.5,0.5)$, and the point $(-0.5,0)$ to $(0.3,-0.1)$.

3 Find the midpoint of the hyperbolic line segment from $(0,0.5)$ to $\left(\frac{-30}{58},\frac{41}{58}\right)$. [You may find the formula $\tanh 2x = \frac{2\tanh x}{1+\tanh^2 x}$ and the result in question 1(a) useful.]

4 Show that if A, B and C are three points in the hyperbolic plane such that $AB = AC$ (as hyperbolic distances). Show that A lies on the perpendicular bisector of the hyperbolic line from B to C. [Hint: apply an isometry to make A the origin, then show that the hyperbolic perpendicular bisector is also the Euclidean perpendicular bisector.]

5 The hyperbolic line through the points $(0,0.5)$ and $(0.5,0)$ is a Euclidean circle. Find the points where this circle meets the boundary of the unit disc.