MATH 2112/CSCI 2112, Discrete Structures I
Winter 2007
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Homework Sheet 1
Solutions and hints

Here are model solutions to the easier questions, and hints for the more difficult questions. For the questions where I have only given hints, you may submit revised solutions for half credit with your solutions to the next assignment. I will give out model solutions to those questions with the solutions and hints for that assignment.

These model solutions do not always list all possible ways of doing the questions.

1 *Rewrite these sentences symbolically:* (Let M="Maths is fun.", L="Dr. Kenney is a good lecturer.", A="I will get an A.", H="I will work very hard.")

(a) Maths is fun but Dr Kenney is not a good lecturer.

$M \wedge \neg L$

(b) If I work very hard then if Dr Kenney is a good lecturer then I will get an A.

$H \rightarrow (L \rightarrow A)$

(c) In order for me to get an A, It is necessary that I work very hard.

$A \rightarrow H$

(d) It is not the case that if I work very hard then maths is fun.

$\neg(H \rightarrow M)$

2 *Which of the following pairs of propositions are logically equivalent? Justify your answers.*

(a) p *and* $(p \rightarrow q) \rightarrow p$

The truth table is:

p	q	$p \to q$	$(p \to q) \to p$
0	0	1	0
0	1	1	0
1	0	0	1
1	1	1	1

So they are equivalent.

(b) $p \wedge \neg q$ *and* $\neg p \to \neg q$

If p and q are both true, then $\neg p \to \neg q$ is true, but $p \wedge \neg q$ is false, so the two propositions are not logically equivalent.

(c) $p \to (q \vee p)$ and $p \vee q$

If p and q are both false, then $p \to (q \vee p)$ is true, but $p \vee q$ is false, so the two propositions are not logically equivalent.

3 Use De Morgan's Laws to write out the negation of the following sentences:
(a) I will work very hard or I will fail.

I will not work very hard, and (or but) I will not fail.

(b) Maths is fun and I will work very hard.

Either maths is not fun or I will not work very hard.

(c) Maths is not fun, and I will not work very hard.

Either maths is fun or I will work very hard.

4 Show the following logical equivalences using the equivalences in 1.1.1.:
(a) $(p \wedge (q \vee \neg q)) \wedge (p \vee (q \wedge \neg q))$ *and* p

$q \vee \neg q$ is a tautology (1.1.1.5), so $p \wedge (q \vee \neg q) \equiv p$ by (1.1.1.4). Similarly, $q \wedge \neg q$ is a contradiction (1.1.1.5), so $p \vee (q \wedge \neg q) \equiv p$ by (1.1.1.4). Therefore, $(p \wedge (q \vee \neg q)) \wedge (p \vee (q \wedge \neg q)) \equiv p \wedge p \equiv p$ by (1.1.1.7).

(b) $q \vee (\neg\neg q \wedge p)$ *and* q

$q \vee (\neg\neg q \wedge p) \equiv q \vee (q \wedge p) \equiv q$ using (1.1.1.6) and (1.1.1.10).

(c) $\neg q \vee (\neg\neg q \wedge p)$ and $\neg q \vee p$

$\neg q \vee (\neg\neg q \wedge p) \equiv \neg q \vee (q \wedge p) \equiv (\neg q \vee q) \wedge (\neg q \vee p) \equiv (q \vee \neg q) \wedge (\neg q \vee p) \equiv \neg q \vee p$ using (1.1.1.6), (1.1.1.3), (1.1.1.1), (1.1.1.5), and (1.1.1.4).

5 *Show that if for any propositions p, q, and r (not necessarily primitive propositions) $p \vee r \equiv p \vee q$ and $p \wedge r \equiv p \wedge q$ then we must have $q \equiv r$.*

Hints:

You want to show $q \equiv r$, given $p \wedge q \equiv p \wedge r$ and $p \vee q \equiv p \vee r$. Start with the equivalences $q \equiv q \vee (q \wedge p) \equiv q \vee (p \wedge q) \equiv q \vee (p \wedge r)$. You still need to use the equivalence $p \vee q \equiv p \vee r$.

Alternatively, use truth tables: Recall that for any propositions s and t, $s \equiv t$ if and only if the proposition $s \leftrightarrow t$ is a tautology.

6 *Using the rules of inference in table 1.3.1, and the logical equivalences in table 1.1.1, show that the following conclusions follow frow the premises given: (State which rule of inference you are using at each step.)*

(a) From $(p \to (p \to p)) \to (p \to p)$ and $p \to (p \to p)$, deduce $p \to p$.

This follows in one step by modus ponens.

(b) From $p \wedge (q \vee r)$, deduce $(p \vee q) \vee s$.

$p \wedge (q \vee r)$	
p	by specialisation
$p \vee q$	by generalisation
$(p \vee q) \vee r$	by generalisation

(c) From $p \to q$ and $(p \to r) \vee (q \to r)$, deduce $p \to r$.

Hint:

It might seem that elimination is a good way to show the result, but this approach won't work because we can't prove $\neg(q \to r)$ (you can see from the truth table that it doesn't follow).

In fact, we need to use a division into cases argument. For this we will have to show $(p \to r) \to (p \to r)$ and $(q \to r) \to (p \to r)$. The first is a tautology. The second will have to be deduced by transitivity from $(q \to r) \to (q \to r)$ and $(q \to r) \to (p \to q)$. The first is a tautology, so we just need to prove $(q \to r) \to (p \to q)$ from $p \to q$.

7 Find Boolean expressions for the following logic circuits:

(a) $(p \wedge q) \vee \neg p$

(b) $(p \vee q) \wedge (\neg q \vee r)$

8 Write the converse and the contrapositive of the following propositions:

(a) If n is prime, then either n is odd, or $n = 2$.

Converse: "If either n is odd or $n = 2$, then n is prime."

Contrapositive: "If it is not the case that either n is odd or $n = 2$, then n is not prime."

(b) If the angle ABC is a right-angle, then AC is a diameter of the circle passing through A, B and C.

Converse: "If AC is a diameter of the circle passing through A, B and C, then the angle ABC is a right angle."

Contrapositive: "If AC is not a diameter of the circle passing through A, B and C, then the angle ABC is not a right angle."