MATH 2112/CSCI 2112, Discrete Structures I

Winter 2007

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Homework Sheet 1

Due in: Wednesday 17th January, 1:30 PM

Compulsory questions

1 Rewrite these sentences symbolically:

(a) Maths is fun but Dr Kenney is not a good lecturer.

(b) If I work very hard then if Dr Kenney is a good lecturer then I will get an A.

(c) In order for me to get an A, It is necessary that I work very hard.

(d) It is not the case that if I work very hard then maths is fun.

2 Which of the following pairs of propositions are logically equivalent? Justify your answers.

(a) p and $(p \to q) \to p$

(b) $p \wedge \neg q$ and $\neg p \to \neg q$

(c) $p \to (q \vee p)$ and $p \vee q$

3 Use De Morgan's Laws to write out the negation of the following sentences:

(a) I will work very hard or I will fail.

(b) Maths is fun and I will work very hard.

(c) Maths is not fun, and I will not work very hard.

4 Show the following logical equivalences using the equivalences in 1.1.1.:

(a) $(p \wedge (q \vee \neg q)) \wedge (p \vee (q \wedge \neg q))$ and p

(b) $q \vee (\neg\neg q \wedge p)$ and q

(c) $\neg q \vee (\neg\neg q \wedge p)$ and $\neg q \vee p$

5 Show that if for any propositions p, q, and r (not necessarily primitive propositions) $p \vee r \equiv p \vee q$ and $p \wedge r \equiv p \wedge q$ then we must have $q \equiv r$.

6 Using the rules of inference in table 1.3.1, and the logical equivalences in table 1.1.1, show that the following conclusions follow frow the premises given: (State which rule of inference you are using at each step.)

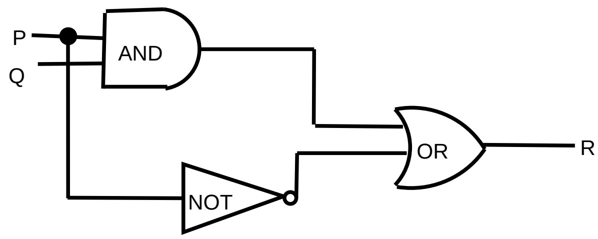
(a) From $(p \to (p \to p)) \to (p \to p)$ and $p \to (p \to p)$, deduce $p \to p$.

(b) From $p \wedge (q \vee r)$, deduce $p \vee q \vee s$.

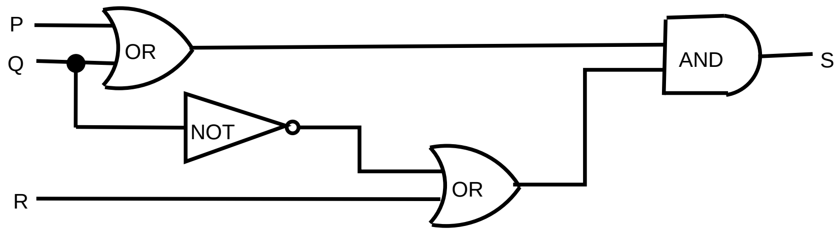
(c) From $p \to q$ and $(p \to r) \vee (q \to r)$, deduce $p \to r$.

7 Find Boolean expressions for the following logic circuits:

(a)



(b)



8 Write the converse and the contrapositive of the following propositions:

(a) If n is prime, then either n is odd, or $n = 2$.

(b) If the angle ABC is a right-angle, then AC is a diameter of the circle passing through A, B, and C.