MATH 2112/CSCI 2112, Discrete Structures I
Winter 2007
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Homework Sheet 10
Due: Wednesday 4th April: 1:30 PM

Compulsory questions

1 For each of the following relations, determine which of the four properties: reflexivity, symmetry, antisymmetry, and transitivity hold for that relation.

(a) The relation on $\mathbb{N}$ that relates two natural numbers if their difference is at most 7.

(b) The relation "is an ancestor of" on the set of all people.

(c) The relation "has the same birthday as" on the set of all people.

(d) The relation "is a square root of" on the set of real numbers.

(e) The relation "is (strictly) taller than" on the set of all people.

(f) The relation on $\mathbb{N}$ that relates two natural numbers if they are p^a and p^b for some prime p and positive integers a and b.

2 How many partial orders are there on a 3-element set (up to rearranging the elements of the set, so for example, $0 < 1 < 2$ and $1 < 0 < 2$ count as the same order)? Give all the corresponding Hasse diagrams.

3 For each of the following functions, determine whether the function is injective, and whether it is surjective. Justify your answers.

(a) $f : \mathbb{R}^+ \to \mathbb{R}$, $f(x) = x^2$.

(b) $f : \mathbb{R} \to \mathbb{R}$, $f(x) = x^2$.

(c) $f : \{0, 1, 2, 3\} \to \{0, 1, 2\}$, $f(0) = 0, f(1) = 2, f(2) = 2, f(3) = 1$.

(d) $f : \mathbb{Q} \to \mathbb{N}$, $f\left(\frac{a}{b}\right) = b$ whenever $(a, b) = 1$.

(e) $f : \mathbb{Z} \to \mathbb{N}$, $f(n) = \begin{cases} 2n & \text{if } n \geqslant 0 \\ -2n-1 & \text{if } n < 0 \end{cases}$

4 Suppose $f : B \to C$ and $g : A \to B$ are functions with composite $f \circ g : A \to C$. Give a proof or a counterexample for each of the following:

(a) If $f \circ g$ is injective then f is injective.

(b) If $f \circ g$ is injective then g is injective.

(c) If $f \circ g$ is surjective then f is surjective.

(d) If $f \circ g$ is surjective then g is surjective.