MATH 2112/CSCI 2112, Discrete Structures I
Winter 2007
Toby Kenney
Homework Sheet 2
Due in: Wednesday 24th January, 1:30 PM

Compulsory questions

1 Which of the following are true when $A = \{0, 1, 2\}$ and $B = \{1, 2, 3, 4\}$? Justify your answers.

(a) $(\forall n \in A)(\exists m \in B)(m = n + 1)$

(b) $(\exists m \in B)(\forall n \in A)(m = n + 1)$

(c) $(\exists n \in B)(n \in A)$

(d) $(\exists m \in B)(\forall n \in A)(n < m)$

(e) $(\forall m \in B)(m + 2 \in A \rightarrow m + 3 \in A)$

(f) $(\exists m \in B)(m + 2 \in A \wedge m + 3 \in A)$

2 Write down the negations of the following statements. Always write them in such a way that the negation acts directly on the predicate, not on any quantifiers.

(a) All students in this course will get "A"s.

(b) Every student will miss at least one lecture. (i.e. For every student, there will be at least one lecture that student misses.)

(c) There will be a lecture that every student attends.

3 Use Venn diagrams to show the following arguments are not valid:

(a)

$$\begin{aligned}
&(\forall x \in A)(x \in B \vee x \in C)\\
&(\forall x \in B)(x \in A \vee x \in C)\\
&\therefore (\forall x \in B)(x \in C)
\end{aligned}$$

(b)

$$\begin{aligned}
&(\forall x \in A)(P(x))\\
&(\forall x \in B)(\neg x \in A)\\
&\therefore (\exists x \in B)(\neg P(x))
\end{aligned}$$

(c)

$$\begin{aligned}
&(\forall x \in A)(\neg x \in B)\\
&(\forall x \in B)(\neg x \in C)\\
&\therefore (\forall x \in A)(\neg x \in C)
\end{aligned}$$

4 Show that the following arguments are valid using universal instantiation and the rules of inference in Chapter 1:

(a)

$$\begin{aligned}&\phi(x)\\&\therefore (\exists y)(\phi(y))\end{aligned}$$

(b)

$$\begin{aligned}&(\forall x \in A)(x \in B)\\&(\exists x \in A)(x \in C)\\&\therefore (\exists y \in B)(y \in C)\end{aligned}$$

(c)

$$\begin{aligned}&(\forall x \in A)(x \in B \to x \in C)\\&(y \in A)\\&y \in C \to \neg(y \in B)\\&\therefore \neg y \in B\end{aligned}$$

(d)

$$\begin{aligned}&(\forall x \in A)(x \in B \vee x \in C)\\&y \in A \wedge \neg(y \in B)\\&\therefore (y \in C)\end{aligned}$$

5 Another quantifier that is sometimes used is $(\exists! x)(\phi(x))$, meaning that there is exactly one x such that $\phi(x)$ is true. Rewrite this expression using $\exists$ and $\forall$ (and any other logical symbols $\wedge$, $\vee$, $\neg$, $\to$, $=$ if necessary).