MATH 2112/CSCI 2112, Discrete Structures I
Winter 2007
Toby Kenney
Midterm Examination
Wednesday 28th February: 6:00-7:30 PM

Answer all questions.

1 Use universal instantiation and rules of inference to show that the following argument is valid.

$$(\forall x \in A)(x \in B)$$
$$\neg((\exists y \in C)(\neg(y \in A)))$$
$$z \in C$$
$$\therefore z \in B$$

2 Which of the following are true when $A = \{0, 2, 5, 7\}$ and $B = \{2, 3, 5, 8, 9, 28\}$? Justify your answers.

(a) $(\forall x \in A)(\exists y \in B)(x \times y \text{ is a perfect square})$

(b) $(\exists y \in B)(\forall x \in A)(x \times y \text{ is a perfect square})$

3 Prove or disprove the following. You may use results proved in the course or the homework sheets, provided you state them clearly.

(a) $\sqrt[3]{4}$ is irrational.

(b) There is a natural number n such that $6n^3 + 12n^2 + 15n + 21$ is prime.

(c) There is a natural number n such that $n^2 + 8n + 6$ is prime.

(d) $n^3 + 5 = m^6 + 9$ has no integer solutions. [Hint: try modulo 7.]

(e) For all natural numbers n, $\sum_{i=1}^{n} \frac{i(i+1)}{2} = \frac{n(n+1)(n+2)}{6}$

(f) There are infinitely many primes congruent to 3 modulo 6.

(g) There are infinitely many prime numbers p such that there is an integer n for which $n^2 \equiv -1 \pmod{p}$. [Hint: Suppose the set of all such prime numbers is $p_1, \ldots, p_k$, and consider $(p_1 p_2 \cdots p_k)^2 + 1$.]

4 Which of the following pairs of propositions are logically equivalent? Justify your answers.

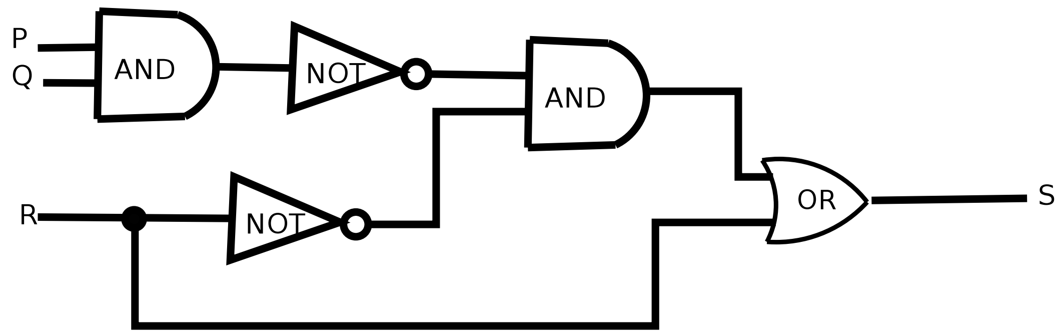
(a) $(p \wedge \neg q) \vee (\neg p \wedge q)$ and $(p \vee q) \wedge \neg(p \wedge q)$.

(b) $p \vee \neg q$ and $\neg(\neg p \vee q)$.

5 Find $0 \leqslant n < 630$ satisfying all the following congruences:

$$n \equiv 3 \pmod{7} \quad (1)$$
$$n \equiv 8 \pmod{10} \quad (2)$$
$$n \equiv 4 \pmod{9} \quad (3)$$

6 Find a boolean expression for the following logic circuit.



7 Use Euclid's algorithm to find the greatest common divisor of the following pairs of numbers. Write down all the steps involved. Use your calculations to find integers a and b such that a times the first number plus b times the second number is their greatest common divisor.

(a) 238 and 133

(b) 289 and 102