MATH 2113/CSCI 2113, Discrete Structures II
Winter 2008
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Homework Sheet 2
Due in: Wednesday 23rd January, 1:30 PM

Compulsory questions

1 Show that $\binom{2n}{n} = \sum_{k=0}^{n} \binom{n}{k}^2$.

2 (a) Show that $\binom{n}{a}\binom{n-a}{b} = \binom{n}{a+b}\binom{a+b}{a}$.

   (b) What is $\sum_{k=1}^{n} \binom{n}{k}k^2$? [Hint: $k^2 = 2\binom{k}{2} + \binom{k}{1}$.]

3 How many subsets of $\{1, 2, \ldots, 17\}$ contain at most two multiples of 3?

4 (a) How many solutions are there to $x_1 + x_2 + x_3 + x_4 = 18$ where x_1, x_2, x_3 and x_4 are natural numbers ($\{0, 1, 2, 3, \ldots\}$)?

   (b) How many solutions are there to $x_1 + 2x_2 + 3x_3 = 10$ for x_1, x_2 and x_3 natural numbers?

5 (a) In a class with 13 students, there are 5 mathematicians and 8 computer scientists. How many subsets of the students in the class contain the same number of mathematicians and computer scientists? [Hint: The easy way to answer this question involves considering a different set from the set to be chosen (but related to it).]

Bonus question

(b) How many contain at least as many mathematicians as computer scientists? [Hint: This question will be a lot easier if you can find a simple explanation for why the solution to part (a) is what it is.]