

MATH 2113/CSCI 2113, Discrete Structures II

Winter 2008

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Homework Sheet 5

Due: Wednesday 13th February: 1:30 PM

**Compulsory questions**

- 1 Let  $a_n$  be defined recursively by  $a_n = \sum_{i=1}^n \frac{a_{n-i}(-1)^{i-1}}{i!}$  for  $n \geq 1$ , and  $a_0 = 1$ .
  - (a) Find the generating function for  $a_n$ .
  - (b) Find a general formula for  $a_n$ .
- 2 (a) Find a recurrence relation for the number of ways to tile a  $3 \times n$  chessboard with  $3 \times 1$  blocks.
  - (b) Find a recurrence relation for the number of ways to tile a  $3 \times n$  chessboard with  $3 \times 1$  and  $3 \times 2$  blocks.
- 3 We have a sequence of  $n$  tiles that we want to paint with 4 colours: red, green, blue and yellow, in such a way that no two adjacent tiles are the same colour, and no two adjacent pairs of tile have the same colours - so for example RGBGB would not be a valid colouring, since the sequence GB is repeated.
  - (a) Find a recurrence relation for the number of valid colourings.
  - (b) Solve your recurrence relation to get an explicit formula.
- 4 Define the sequence  $a_n$  recursively by  $a_0 = 0$  and  $a_{n+1} = 2a_n + (n+1)^2$  for  $n \geq 0$ .
  - (a) Find the generating function for  $a_n$ .
  - (b) Expand the generating function as a partial fraction.
  - (c) Use this to find the  $a_n$ .