# MATH 2113/CSCI 2113, Discrete Structures II
## Winter 2008
## Toby Kenney
### Homework Sheet 6
### Due: Wednesday 20th February: 1:30 PM

## Compulsory questions

1 We have 10 points in a square of side 3cm. Show that two of the points are within $\sqrt{2}$cm of each other.

2 (a) You are dealt 13 cards. You may choose 5 of them to make a poker hand. Assuming you make the best poker hand possible from the 13 cards, what is the worst hand you can end up with (and why)? [The type of hand is sufficient – you don't need to give the exact cards.]

(b) What if you are dealt 17 cards? [You still have to create a 5-card hand.]

3 (i) Find the generating function for $\binom{n}{k}k^2$ as a function of $k$. [Hint: You know the generating function for $\binom{n}{k}$.]

(ii) Deduce that $\sum_{k=1}^{n} \binom{n}{k}k^2 = 2^{n-2}n(n+1)$. [Recall that we also showed this in a different way on sheet 2.]

4 At a party, there are $n$ people. Each of them knows some number of the other people. Show that there are some two people at the party who know the same number of other people. [You may assume that if $A$ knows $B$ then $B$ also knows $A$.]

5 I have a sequence of integers $a_1, a_2, \ldots, a_n$ with the property that for any $i \leqslant j$, $a_i + a_{i+1} + \ldots + a_j$ is not a multiple of 17. What is the largest possible value of $n$. Justify your answer.

6 There are 100 students in a class. There are 9 homework sheets.

(a) Suppose that for each homework sheet at least $n$ students hand in solutions. How large does $n$ have to be to ensure that there is some student who attempts all the homework sheets?

(b) Suppose now that each student submits solutions to at least $m$ of the homework sheets. How large must $m$ be to ensure that there are 2 students who submit solutions to the same set of homework sheets?