MATH 2113/CSCI 2113, Discrete Structures II
Winter 2008
Toby Kenney
Mock Final Examination
Time allowed: 3 hours

Justify all your answers. There are deliberately more than 3 hours worth of questions here to give you a wider variety of questions. There are also more questions on the later half of the course, and particularly on Ramsey theory than there are likely to be in the final exam.

Compulsory questions

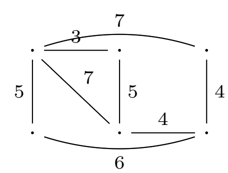
1 Deduce the finite version of Ramsey's theorem from the infinte version.

2 A biassed coin with probability 0.3 of getting a head is tossed 50 times. What is the expected number of occurences of the sequence "HTTHH"?

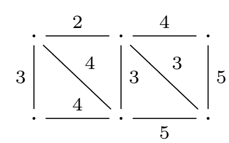
3 How many non-identical ways are there to colour the sides of a square with 3 colours, where we count rotations as identical, but not reflections.

4 Find minimal spanning trees for the following graphs:

(a)

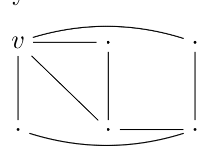


(b)



5 A fair die is rolled repeatedly until a 6 is rolled. What is the expected number of rolls required before a 6 is rolled?

6 (a) Write down the adjacency matrix for the graph:



(b) How many walks of length 8 are there starting and ending at v?

(c) Does the graph have an Euler Circuit?

(d) Does it have a Hamiltonian cycle?

7 A fair die is rolled, and the result is used to select a number of coins to toss – e.g. if a 3 is rolled, we toss 3 coins. What is the probability that the die roll was a 4 given that there were exactly 2 heads among the coins tossed?

8 Show that any 2-colouring of a K_6 actually has 2 monochromatic triangles. [Hint: we know it must have one. Let it be v_1, v_2, v_3, and w.l.o.g., let it be red. Consider cases:

1. There are no red edges wv_i where w is not one of the v_i.

2. There is a w not one of the v_i with exactly one red edge wv_i, and the two edges from w to vertices outside the triangle are both red.

3. There is a w not one of the v_i with exactly one red edge wv_i, and one of the two edges from w to a vertex outside the triangle is blue.

4. There are red edges wv_i and wv_j for w not one of the v_i.]

9 (a) Show that whenever we 4-colour the edges of a K_{66}, we always get a monochromatic triangle.

(b) What is the expected number of monochromatic triangles?

10 How many 4-digit numbers counting numbers with leading zeros, contain at least one of the digits 1 and 2.

11 If we 2-colour a K_n, must there be a monochromatic spanning tree?

(b) What if we 3-colour it?

12 3 fair dice are rolled. One is red; one is green and one is blue.

(a) What is the probability that red $\leqslant$ green $\leqslant$ blue.

(b) What is the probability that blue $<$ green given that red $\leqslant$ green?

13 Show that at least 4 trees are required to cover all edges of K_8?

14 Two players play a game: player A rolls a fair die and scores the result of the roll. Player B tosses 6 fair coins and records the number of heads. Player B wins if his score is greater than or equal to A's score. What is the probability that B wins.

15 Let p_n be the probability of tossing a fair coin n times without getting 4 consecutive heads. Show that $p_n = \frac{1}{2}p_{n-1} + \frac{1}{4}p_{n-2} + \frac{1}{8}p_{n-3} + \frac{1}{16}p_{n-4}$.