MATH 2600/STAT 2600, Theory of Interest
FALL 2010
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Homework Sheet 1
Due: Thursday 30th September: 1:00 PM

1. *Calculate the accumulated value on maturity of the following investments:*

   (a) \$1,000 invested for 3 months at 5% simple interest.

$$1000 \times (1 + \frac{3}{12} \times 0.05) = 1012.50$$

   (b) \$200 invested on 16th August 2007 and maturing on 4th March 2008, at 7% exact simple interest. (NB. 2008 is a leap year.)

   4th March is day 63+1=64.

   16th August is day 228

   Term of loan = 64+365-228=201 days.

   Accumulated value=

$$200 \times (1 + \frac{201}{365} \times 0.07) = 207.71$$

   (c) \$1,000,000 invested on 12th May 2009 and maturing on 6th June 2009 at 6% ordinary simple interest.

   6th June is day 157

   12th May is day 132

   Term of loan = 157-132=25 days.

$$1000000 \times (1 + \frac{25}{360} \times 0.06) = 1004166.67$$

2. *A man invests \$1,000 at 12% exact simple interest on 7th March.*

   (a) On what date does he have enough money to buy a computer which costs \$1,100 (including taxes)?

   We want to find the day on which

$$1000 \times (1 + t \times 0.12) \geqslant 1100$$

   This is equivalent to

$$1000 \times 0.12 \times t \geqslant 100$$

   or

$$t \geqslant \frac{100}{120}$$

Now the number of days is given by $365 \times t$, so we get a total of $365 \times \frac{100}{120} = 304.17$ days. The smallest whole number of days that is at least this much is 305.

(b) How much money does he have left over if he buys the computer on that day?

After 305 days, the man has

$$1000 \times (1 + \frac{305}{365} \times 0.12) = 1100.27$$

so after buying the computer, he has 27 cents left over.

3. *A businessman receives an invoice for \$700, with terms 2/10 n/50. What is the largest rate of interest at which it is worth his taking out a loan to get the discount.*

   He can either pay \$700 in 50 days, or pay \$686 in 10 days. So it is worth taking out the loan if the accumulated value of \$686 after 40 days is less than \$700. That is if

$$686 \times (1 + \frac{40}{365} \times r) < 700$$

   This is equivalent to

$$686 \times \frac{40}{365} \times r < 14$$
$$r < \frac{14 \times 365}{40 \times 686} = 0.1862\ldots$$

   So it is worth taking out the loan at an interest rate of 18.62% or less.

4. *An investor has a promissory note for \$10,000 in 240 days at 8% simple interest. After 230 days, she sells it to a bank which discounts notes at 7.8% interest.*

   (a) How much does the bank pay for the note?

   The legal due date of the note is in 243 days. At that time, the accumulated value of the note is

$$10000 \times (1 + \frac{243}{365} \times 0.08) = 10532.60$$

   The bank buys the note 13 days before maturity, so the price the bank pays is

$$\frac{10532.60}{1 + \frac{13}{365} \times 0.078} = 10503.42$$

   (b) What rate of return does the investor achieve?

The investor makes a profit of \$503.42, on a principal of \$10,000 in 230 days, so the interest rate achieved is

$$\frac{503.42}{10000 \times \frac{230}{365}} = 0.0799$$

i.e. 7.99%

(c) If the debtor pays the debt on the due date (i.e., not after 3 days grace), what rate of return does the bank achieve?

If the debtor pays the debt after 240 days, the amount due is

$$10000 \times (1 + \frac{240}{365} \times 0.08) = 10526.03$$

Therefore, the bank makes a profit of \$22.61 in 10 days, on an investment of \$10503.42, so its rate of return is

$$\frac{22.61}{10503.42 \times \frac{10}{365}} = 0.0786$$

or 7.86%

5. *Mr. Smith takes out a loan of \$10,000 at 5% simple interest on 11th May 2011. He pays it back with a repayment x on 12th July 2011, and a repayment of \$4,000 on 3rd September 2011. The loan is calculated using exact interest.*

   11th May is day 131

   12th July is day 193

   3rd September is day 246

   (a) Write down and solve an equation of value for x with focal date:

   (i) 11th May.

   On 11th May, the value of the loan is \$10,000. The value of the first repayment is

$$\frac{x}{1 + \frac{62}{365} \times 0.05}$$

   The value of the second repayment is

$$\frac{4000}{1 + \frac{115}{365} \times 0.05} = 3937.9636$$

   The equation of value therefore says

$$\begin{aligned} 10000 &= \frac{x}{1 + \frac{62}{365} \times 0.05} + 3937.9636 \\ 6062.0364 &= \frac{x}{1 + \frac{62}{365} \times 0.05} \\ x &= 6062.0364 \times (1 + \frac{62}{365} \times 0.05) \\ &= 6113.52 \end{aligned}$$

(ii) 12th July.

On 12th July, the value of the loan is

$$10000 \times (1 + \frac{62}{365} \times 0.05) = 10084.93$$

The value of the first repayment is x. The value of the second repayment is

$$\frac{4000}{1 + \frac{53}{365} \times 0.05} = 3971.17$$

The equation of value therefore says

$$\begin{aligned} x + 3971.17 &= 10084.93 \\ x &= 6113.76 \end{aligned}$$

(iii) 3rd September.

On 3rd September, the value of the loan is

$$10000 \times (1 + \frac{115}{365} \times 0.05) = 10157.53$$

The value of the first repayment is

$$x(1 + \frac{53}{365} \times 0.05)$$

The value of the second repayment is $4,000. The equation of value therefore says

$$\begin{aligned} 10157.53 &= x(1 + \frac{53}{365} \times 0.05) + 4000 \\ x(1 + \frac{54}{365} \times 0.05) &= 6157.53 \\ x &= \frac{6157.53}{1 + \frac{53}{365} \times 0.05} \\ &= 6113.15 \end{aligned}$$

(b) Which (if any) of these gives the correct value for x?

It depends on the method agreed for crediting partial payments. Using the declining balance method is equivalent to (ii), while using the merchant's rule is equivalent to (iii).

[Note that when there are multiple partial payments, the declining balance method is no longer equivalent to an equation of value at a single focal date.]

6. *A man takes out a one-year loan for \$10,000 at 7% simple interest. After 4 months, he repays \$2,500. After 7 months from the start of the loan, he repays a further \$4,000.*

   (a) Calculate the outstanding balance at the end of the year, using:

   (i) The Declining Balance Method.

   After 4 months, the accumulated value is

$$10000 \times (1 + \frac{4}{12} \times 0.07) = 10233.33$$

   so after the \$2,500 payment, the balance is \$7733.33. Now 3 months later, the accumulated value is

$$7733.33 \times (1 + \frac{3}{12} \times 0.07) = 7868.66$$

   so after the \$4,000 payment, the balance is \$3,868.66. The end of the loan is 5 months later, so the balance is

$$3969.66 \times (1 + \frac{5}{12} \times 0.07) = 4085.44$$

   (ii) The Merchant's Rule.

   The accumulated value of the loan is

$$10000 \times (1 + 1 \times 0.07) = 10700$$

   The accumulated value of the first repayment is

$$2500 \times (1 + \frac{8}{12} \times 0.07) = 2616.67$$

   The accumulated value of the second repayment is

$$4000 \times (1 + \frac{5}{12} \times 0.07) = 4116.67$$

   so the outstanding balance is \$3966.66

   (b) Suppose that the rule for partial payments is the Declining Balance Method. If the borrower can invest his money in a bank account, and receive an interest rate of r simple interest, what is the smallest value of r, such that it is better to do this than to make the repayments above?

   If the borrower invests his partial payments at a rate r of simple interest, then the total amount he will have at the end of the year is

$$2500 \times (1 + \frac{8}{12}r) + 4000 \times (1 + \frac{5}{12}r) = 6500 + 3333.33r$$

   Therefore, the amount of additional money needed to pay off the loan is obtained by subtracting this from \$10,700, i.e $4,200 - 3333.33r$. For this method to be better than paying off the loan, we need this amount of extra money needed to be less than \$4085.44, or equivalently, we need $3333.33r > 114.56$, which is equivalent to $r \geqslant 0.0344$, i.e. if r is at least 3.44%.

7. *What rate of simple discount is equivalent to 11% simple interest over a period of 9 months?*

   For 11% simple interest over 9 months, the accumulation factor is $(1 + \frac{9}{12} \times 0.11) = 1.0825$. To obtain a simple discount whose accumulation factor is 1.0825 over 9 months, we need to solve

$$
\begin{aligned}
\frac{1}{1-\frac{9}{12}r} &= 1.0825 \\
1-\frac{9}{12}r &= \frac{1}{1.0825} \\
r &= \frac{12}{9}\left(1-\frac{1}{1.0825}\right) \\
&= 0.1016
\end{aligned}
$$

   So the equivalent rate is 10.16%