MATH 2600/STAT 2600, Theory of Interest
FALL 2010
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Homework Sheet 6
Model Solutions

1. *Calculate the price that should be paid for each of the following bonds to obtain the desired yield:*

   (a) face value \$5,000, maturing at par in 15 years, coupon rate 8%, desired yield $j_2 = 7\%$.

   $5000 + 5000(0.04 - 0.035)\frac{1-1.035^{-30}}{0.035} = 5459.80$

   (b) face value \$2,000, maturing at 110 in 5 years, coupon rate 6%, desired yield $j_2 = 6.5\%$.

   $2200 + (60 - 2200 \times 0.0325)\frac{1-1.0325^{-10}}{0.0325} = 2103.14$

2. *Calculate the yield from the following bond, using linear interpolation to get a more accurate answer. Face value \$1,000, maturing at par in 5 years, coupon rate 4%, bought for \$922*

   method of averages gives:

   $\frac{10\times20+1000-922}{1922\times5} = 2.892...\%$ every half-year.

   We try 2.9% every half year: 922.83... We try 3% every half year: 914.697...

   interpolating, we get: 2.91%..., or $j_2 = 5.82\%$.

3. *Write out a complete bond amortisation schedule for a bond with face value \$2,000 with coupon rate 5%, maturing at 105 in 4 years, sold to an investor who wishes to receive a yield of 6%.*

   The selling price is given by $2100 + (50 - 63)\frac{1-1.03^{-8}}{0.03} = 2008.74$.

Coupon Interest	Interest on Book Value	Book Value Adjustment	Book Value
			2008.74
50	60.26	-10.26	2019.01
50	60.57	-10.57	2029.58
50	60.89	-10.89	2040.46
50	61.21	-11.21	2051.68
50	61.55	-11.55	2063.23
50	61.90	-11.90	2075.12
50	62.25	-12.25	2087.38
50	62.62	-12.62	2100.00

4. *An investor wants to buy a callable bond with face value \$10,000, maturing at par in 10 years, with coupon rate 6%, callable at 105 after 5 years.*

   (a) If she wishes to guarantee a return of 7%, how much should she pay?

If the bond is uncalled, she would pay $10000 + (300 - 350)\frac{1-1.035^{-20}}{0.035} = 9289.38$

If it is called after 5 years, she should pay $10500+(300-367.50)\frac{1-1.035^{-10}}{0.035} = 9938.63$

so she should pay \$9,289.38.

(b) If she sells it after 3 years to an investor who wishes to guarantee a return of 4%, how much does she receive?

If the bond is uncalled, the value to the investor is

$10000 + (300 - 200)\frac{1-1.02^{-14}}{0.02} = 11210.62$,

If it is called, the value is

$10500 + (300 - 210)\frac{1-1.02^{-4}}{0.02} = 10842.70$.

She would receive the lower of these, i.e. \$10,842.70.

(c) If the bond were not callable after 5 years, for how much would she be able to sell it to the buyer in (b)?

\$11,210.62

5. *A bond with face value \$500, maturity date 1st January 2033 and coupon rate 5% is sold on 4th March 2011 to an investor who wishes a yield of 4.5%. Calculate*

   (a) The flat price [use compound interest].

   The price on 1st January 2011 would be

   $500+500(0.025-0.0225)\frac{1-1.0225^{-44}}{0.0225} = 534.68...$, so the price on 4th March, 62 days later, is $534.68... \times 1.0225^{\frac{62}{181}} = 538.77....$

   (b) The quoted price.

   The accumulated interest is $12.50\frac{62}{181} = 4.28...$, so the quoted price is \$534.49.

6. *An investor buys a bond with face value \$1,000, maturing at par in 15 years, coupon rate 5%, for a price to yield 4%. He also buys a strip bond with face value \$2,000, maturing in 15 years, for a price to yield 4%.*

   (a) How much does he pay for each of these bonds?

   For the bond, he pays $1000 + 5\frac{1-1.02^{-30}}{0.02} = 1111.98$, for the strip bond he pays $2000 \times 1.02^{-30} = 1104.14$

   (b) 2 years later, interest rates drop, and he sells both bonds to investors who wish to receive a yield of 3.2%. What prices does he receive for them?

   For the bond, he receives $1000 + 1000(0.025 - 0.016)\frac{1-1.016^{-26}}{0.016} = 1190.21$. For the strip bond, he receives $2000 \times 1.016^{-26} = 1323.71$.

7. *A company are considering a project. The project has the following expected cash flows (all amounts are calculated at the beginning of the year):*

Year	*Net Cash Flow*
0	*-100,000*
1	*-10,000*
2	*30,000*
3	*40,000*
4	*40,000*
5	*40,000*
6	*25,000*
7	*40,000*
8	*40,000*
9	*40,000*
10	*-50,000*

(a) *What is the Net Present value of the project at* $j_1 = 5\%$*?*

$-100000 - 10000 \times 1.05^{-1} + 30000 \times 1.05^{-2} + 40000 \times 1.05^{-3} + 40000 \times 1.05^{-4} + 40000 \times 1.05^{-5} + 25000 \times 1.05^{-6} + 40000 \times 1.05^{-7} + 40000 \times 1.05^{-8} + 40000 \times 1.05^{-9} - 50000 \times 1.05^{-10} = 85734.63$

So the Net Present Value is \$85,734.63.

(b) *What is the Net Present value of the project at* $j_1 = 15\%$*?*

$-100000 - 10000 \times 1.15^{-1} + 30000 \times 1.15^{-2} + 40000 \times 1.15^{-3} + 40000 \times 1.15^{-4} + 40000 \times 1.15^{-5} + 25000 \times 1.15^{-6} + 40000 \times 1.15^{-7} + 40000 \times 1.15^{-8} + 40000 \times 1.15^{-9} - 50000 \times 1.15^{-10} = 20979.51$

So the Net Present value is \$20,979.51.

(c) *What is the internal rate of return?*

We can try a few different values, and eventually interpolate between $j_1 =$ 19%, for a net present value of \$3392.59..., and $j_1 = 20\%$ for a net present value of \$-471.19... This gives an estimate of $\frac{j_1-19}{20-19} = \frac{-3392.59...}{-471.19...-3392.59....}$, which gives $j_1 = 19.88\%$ as the internal rate of return.