MATH 2600/STAT 2600, Theory of Interest
FALL 2013
Toby Kenney
Practice Final Examination

This Sample Examination has more questions than the actual final, in order to cover a wider range of questions. Estimated times are given after each question to help in preparation.

1. Calculate the price that should be paid for each of the following bonds to obtain the desired yield:

   (a) Face value \$80,000, maturing at par in 12 years, coupon rate $j_2 = 7\%$, desired yield $j_2 = 5\%$. [5 mins]

   (b) Face value \$130,000, maturing at par in 35 years, coupon rate $j_2 = 3\%$, desired yield $j_2 = 5\%$. [5 mins]

2. (a) Write out a complete bond amortisation schedule for a bond with face value \$10,000 with coupon rate $j_2 = 2\%$, maturing at par in 2 years, sold to an investor who wishes to receive a yield of $j_2 = 6\%$ [10 mins]

   (b) Write out a complete bond amortisation schedule for a bond with face value \$20,000 with coupon rate $j_2 = 6\%$, maturing at par in 3 years, sold to an investor who wishes to receive a yield of $j_2 = 2\%$ [10 mins]

3. A bond has face value \$30,000, maturity in 20 years, coupon rate $j_2 = 6\%$. After 2 years and 4 months, it is sold to Mr. Armstrong, who wishes to receive a yield of $j_2 = 4\%$. Calculate

   (a) The flat price. [5 mins]

   (b) The quoted price. [5 mins]

4. Mr. Bruce buys a zero-coupon bond with face value \$18,000, maturing at par in 9 years, for a price to yield 4.7% annually. After three years, interest rates increase, and he sells the bond to an investor who wishes to receive a yield of 5.2% annually. What is Mr. Bruce's rate of return? [5 mins]

5. A company are considering a project. The project has the following expected cash flows (all amounts are calculated at the beginning of the year):

Year	0	1	2	3	4	5
Net Cash Flow (000)	-100	10	20	20	10	50

   (a) What is the Net Present value of the project at $j_1 = 2\%$? [5 mins]

   (b) What is the Net Present value of the project at $j_1 = 4\%$? [5 mins]

(c) Which of the following is the internal rate of return? (Justify your answer.) [10 mins]

(i) 2.54% (ii) 2.68% (iii) 2.72% (iv) 5.10%

6. A company are considering two projects. The projects have the following expected cash flows (all amounts are calculated at the beginning of the year):

Year	0	1	2	3	4
Project 1 Net Cash Flow (000)	-150	20	40	70	90
Project 2 Net Cash Flow (000)	-110	40	40	40	40

(a) Which project should they prefer if the cost of capital is $j_1 = 2\%$? [10 mins]

(b) Which project should they prefer if the cost of capital is $j_1 = 12\%$? [10 mins]

7. Mrs. Chapman has $300,000 in her investment fund at the start of the year. One month later, she withdraws $140,000. Another 3 months later (4 months from the start of the year), she withdraws a further $90,000 (there is enough money in the fund to cover this withdrawl). After another 5 months, (9 months from the start of the year), she invests $160,000 in the fund. At the end of the year, she has $395,000 in her account. What is her dollar-weighted rate of return for the year? [10 mins]

8. Mr. Davies is managing an investment fund. At the start of the year, there is $8,000,000 in the fund. The fund activity is summarised in the following table:

Months from start of year	Fund value before transaction	Net Deposit	Fund value after transaction
0			8,000,000
1	7,920,000	-1,000,000	6,920,000
3	7,150,000	500,000	7,650,000
4	7,720,000	100,000	7,820,000
5	7,830,000	600,000	8,430,000
6	8,660,000	-400,000	8,260,000
8	7,840,000	500,000	8,340,000
10	8,220,000	-100,000	8,120,000
12	8,400,000		8,400,000

Calculate his time-weighted rate of return for the year. [10 mins]

9. A company has an account which pays interest at $j_1 = 3\%$ on credit balances, and charges interest at $j_1 = 5\%$ on debit balances. The company is considering using this account to fund possible projects with the following expected cashflows: (All amounts are calculated at the begining of the year.)

Year	0	1	2	3	4	5
Net Cash Flow (000)	-150	30	50	90	50	20

How much money does the company have in its account at the start of the fifth year, when it receives the final cash-flow, if it invests in this project? [10 mins]

10. The current term structure has the following semi-annual yields on zero-coupon bonds:

Term(years)	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3	$3\frac{1}{2}$	4
rate	6%	6.2%	6.2%	5.8%	5.3%	5.1%	4.8%	4.7%

How much should be paid for a $100 face-value bond with semi-annual coupons, maturing at par in 4 years time, with 6% annual coupon rate? [10 mins]

11. For the term structure in Q. 10, what is the implied forward rate for an 18-month loan starting in 1 year's time? [5 mins]

12. The spot-rates for 1, 2 and 3 year strip bonds are 4%, 4.3% and 4.2% annually. You have the opportunity to borrow or lend money at these rates, and you also have the opportunity to arrange to borrow money in two year's time for one year, at an annual rate of 4.4%, or to lend money in two years time for one year at an annual rate of 4.2%. Can you construct an arbitrage possibility? [8 mins]

13. Mr and Mrs. Eastman are borrowing $70,000 at a variable rate of prime+2.4%. They are making interest-only payments annually. They make a forward rate agreement with the bank, so that the interest rate for the third year (starting two years from now) will be 6%. In two years time, the prime rate is 4%. How much money changes hands at the end of the third year, and in which direction? [10 mins]

14. Mrs. Funk can borrow at 8% on the fixed-rate market, or at prime+6.5% on the variable rate market. Dr. Green can borrow at 5% on the fixed-rate market, or at prime+2.9% on the variable rate market. Mr. Harris offers to arrange interest rate swaps with both of them so that Mrs. Funk can borrow $200,000 at prime+6.3% and Dr. Green can borrow $200,000 at 4.8%. How much spread income does Mr. Harris make on this transaction? [10 mins]

15. The current term structure has the following annual yields on zero-coupon bonds:

Term(years)	1	2	3	4	5	6
rate	4.2%	4.1%	4.3%	4.5%	4.6%	4.8%

Mr and Mrs. Ive have a floating rate loan of $600,000, with annual interest-only payments. They wishes to exchange this for a fixed rate over the next 6 years (i.e. They want to pay the same interest rate over the next 6 years). What should this rate be? [10 mins]

16. For the spot rates from Q. 15, which of the following is the at-par yield of a 6-year bond with annual coupons? [10 mins]

(i) 4.60% (ii) 4.68% (iii) 4.72% (iv) 4.76%

17. Calculate the modified duration and Macauley duration of a 10-year bond with semi-annual coupons at coupon rate 8%, if it is purchased for a yield of 6%. [10 mins]

18. A company expects to pay $2,000,000 in 4 years time, and $4,000,000 in 6 years time. If the current spot rates are as in the following table:

Term(years)	2	4	6	9
rate	3.7%	4.3%	4.6%	4.8%

find a way for the company to Reddington immunise these cash-flows by buying zero-coupon bonds with maturities in 2 or 9 years. [15 mins]

19. Explain the difference between full immunisation and Reddington immunisation. [5 mins]

20. Find two payments in 5 and 16 years that immunise the liabilities of a 15-year bond with face value $120,000 and semi-annual coupon rate 7% at a price to yield $j_2 = 5\%$. [15 mins]

21. The current term structure has the following yields on zero-coupon bonds:

Term(years)	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3
rate	6.3%	6.5%	7%	7.1%	7.1%	7.2%

Calculate the modified duration of an 8% semi-annual 3-year bond, based on a parallel shift in the term structure. [15 mins]

Formulae

Macauley duration of a coupon bond:

$$D = \frac{1+i}{i} - \frac{1+i+n(r-i)}{r\left((1+i)^n - 1\right) + i}$$

Increasing annuities:

$$(Ia)_{\overline{n}|i} = \frac{(1+i)^{-1}a_{\overline{n}|i} - n(1+i)^{-n}}{i}$$