MATH 3030, Abstract Algebra
FALL 2012
Toby Kenney

Homework Sheet 12

Due: Monday 11th February: 3:30 PM

Basic Questions

1. Show that it is not possible to trisect an angle of $\cos^{-1}(0.6)$. [An angle of $\cos^{-1}(0.6)$ is constructable.]
2. Show that $x^3 + 2x^2 + 4x + 3$ has distinct zeros in the algebraic closure of $\mathbb{Z}_5$.
3. How many primitive 15th roots of unity are there in GF(16)?
4. Find a basis for the field extension $\mathbb{Q}(\sqrt{2}, \sqrt[3]{3})$ over $\mathbb{Q}$.

Theoretical Questions

5. Let E be algebraically closed, and let F be a subfield of E. Show that the algebraic closure of F in E is also algebraically closed. [So for example, the field of algebraic numbers (that is, complex numbers that are algebraic over $\mathbb{Q}$) is algebraically closed.
6. Let F be a field. Let α be transcendental over F. Show that any element of $F(\alpha)$ is either in F or transcendental over F.
7. Is it possible to duplicate a cube if we are given a unit line segment and a line segment of length $\sqrt[3]{3}$?
8. Show that every irreducible polynomial in $\mathbb{Z}_p[x]$ divides $x^{p^n} - x$ for some n.
9. Show that a finite field of p^n elements has exactly one subfield of p^m elements for any m which divides n.

Bonus Questions

10. Let F_q be the finite field with q elements.

    (a) Show that an irreducible polynomial of degree m in $F_q[X]$ divides $x^{q^n} - x$ if and only if m divides n.

(b) If $a_n(q)$ is the number of irreducible polynomials of degree n over F_q, show that

$$\sum_{d|n} d a_d(q) = q^n$$

(c) How many irreducible polynomials of degree 6 are there over $\mathbb{Z}_3$.