

MATH 3030, Abstract Algebra
FALL 2012
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Homework Sheet 13
Due: Friday 15th February: 3:30 PM

Basic Questions

1. Compute a composition series for S_4 .
2. Let $G = \mathbb{Z}_{30}$, let $K = \langle 6 \rangle$ and let $H = \langle 3 \rangle$. Give an explicit description of the isomorphism $G/H \longrightarrow (G/K)/(H/K)$.
3. In the group $G = S_4$, let $N = \{e, (12)(34), (13)(24), (14)(23)\}$, and let H be the subgroup of permutations that fix 1. Describe the isomorphism between $(HN)/N$ and $H/(H \cap N)$.
4. Let $\phi : \mathbb{Z}_{15} \longrightarrow \mathbb{Z}_5$ be given by $\phi(1) = 3$. Let K be the kernel of ϕ . Explicitly describe the isomorphism given by the isomorphism theorem, between \mathbb{Z}_{15}/K and \mathbb{Z}_5 .

Theoretical Questions

5. Let H and K be subgroups of G , with K normal in G , and such that $HK = G$ and $H \cap K = \{e\}$. Show that $G/K \cong H$.
6. Show that the direct product of two solvable groups is solvable.
7. Show that a subgroup of a solvable group is solvable.

Bonus Questions

8. Show that the homomorphic image of a solvable group is solvable.