MATH 3030, Abstract Algebra
Winter 2013
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Homework Sheet 17
Due: Friday 5th April: 3:30 PM

Basic Questions

1. (a) Is the regular 120-gon constructable?

   (b) Is the regular 28-gon constructable?

   (c) Is the regular 100-gon constructable?

2. Show that if m and n are distinct, and not divisible by p, then $\Phi_m(x)$ and $\Phi_n(x)$ have no common factor in $\mathbb{Z}_p[x]$.

3. (a) Let K be the splitting field of the polynomial $f(x) = x^3 + x^2 + 2$ over $\mathbb{Z}_3$. Is K a radical extension of $\mathbb{Z}_3$?

   (b) is $f(x)$ solvable by radicals over $\mathbb{Z}_3$?

4. Find $\Phi_{12}(x)$ over $\mathbb{Q}$.

Theoretical Questions

5. Show that for a field F of characteristic not dividing n, we have $x^n - 1 = \Pi_{d|n}\Phi_d(x)$. [The product is over all divisors of n.]

6. Show that $f(x) = x^5 - 9x + 6$ is not solvable by radicals over $\mathbb{Q}$.

7. Let K be a normal extension of F with $[K : F] = 26$. Show that K is contained in an extension of F by radicals. [You may assume that any group of order 26 contains an element of order 13, and that any extension with a solvable Galois group is contained in an extension by radicals.]

8. Let f be an irreducible cubic polynomial in $\mathbb{Q}$ with only one real root. Show that the Galois group of f is S_3.