MATH 3030, Abstract Algebra
FALL 2012
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Homework Sheet 6
Due: Friday 2nd November: 3:30 PM

Basic Questions

1. Show that A_5 is simple. [Hint: the conjugacy classes of A_5 have sizes 1, 12, 12, 15 and 20].
2. (a) Calculate the commutator subgroup of D_6.
   (b) Calculate the factor group of D_6 over its commutator subgroup.
3. Calculate the centre of D_8.
4. If H is an abelian normal subgroup of G, must H be contained in $Z(G)$? Give a proof or a counterexample.

Theoretical Questions

5. Show that if G is a simple group, and $G \xrightarrow{\phi} H$ is a homomorphism of G onto H, then either H is trivial, or ϕ is an isomorphism.
6. Let $G \xrightarrow{\phi} A$ be a homomorphism from G to an abelian group, and let C be the commutator subgroup of G. Show that there is a homomorphism $G/C \xrightarrow{\phi'} A$, such that ϕ is the composite $G \longrightarrow G/C \xrightarrow{\phi'} A$.
7. Show that if $G/Z(G)$ is cyclic, then G is abelian.
8. Show that the group of inner automorphisms of a group G is a normal subgroup of the group of all automorphisms of G.
9. Let $N \leqslant H \leqslant G$, and N be a normal subgroup of G.
   (a) Show that N is a normal subgroup of H.
   (b) Suppose H/N is a normal subgroup of G/N. Show that H is a normal subgroup of G.

Bonus Questions

10. Give an example of a group whose commutator subgroup is non-trivial and abelian.