MATH 3030, Abstract Algebra
Winter 2012
Toby Kenney
Midterm Examination
Monday 18th February: 2:35-3:25 PM

Basic Questions

1. Let $R = \mathbb{Z}_4 \times \mathbb{Z}_2$. Let I be the ideal of R generated by $(2, 1)$.

   (a) What is the ideal I?

   (b) What is the factor ring R/I?

2. What is $\mathrm{Irr}(\sqrt{3} + \sqrt{5}, \mathbb{Q})$?

3. Let α be a zero of $f(x) = x^2 - 2$ in GF(25). Find a generator of the multiplicative group of nonzero elements of GF(25). [Write the generator as a polynomial in α.]

4. Compute a composition series for $D_5 \times D_4$. Is $D_5 \times D_4$ solvable?

Theoretical Questions

5. Prove that for a field F, every ideal in the polynomial ring $F[x]$ is principal.

6. Show that any finite extension field E of a field F is algebraic over F.

7. Show that any non-zero ring homomorphism between two fields is one-to-one.

8. Let F be a field. Let $F(\alpha)$ be algebraic over F.

   (a) Show that if $[F(\alpha) : F]$ is odd, then $F(\alpha^2) = F(\alpha)$.

   (b) [Bonus] If $[F(\alpha) : F]$ is not divisible by 3, must $F(\alpha^3) = F(\alpha)$? [Give a proof or a counterexample.]