MATH/STAT 3360, Probability
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Formula Sheet

Combinatorial Analyis

- $n! = n \times (n-1) \times \cdots \times 2 \times 1$ — Number of ways to order n things.
- ${}_nP_m = n(n-1)\cdots(n+1-m)$
- $\binom{n}{m} = \frac{n(n-1)\cdots(n+1-m)}{m!}$
- $\binom{n}{n_1, n_2, \ldots, n_k} = \frac{n!}{n_1!n_2!\cdots n_k!}$ – Number of distinct ways to order n things of k distinct types with n_i of the ith type.

Axioms of Probability

- $0 \leqslant P(E) \leqslant 1$
- $P(S) = 1$.
- If $A_1, A_2, \ldots$ are mutually exclusive, then $P(A_1 \cup A_2 \cup \cdots) = \sum_{i=1}^{\infty} P(A_i)$.

Discrete Random Variables

Distribution	Parameters	$P(X = i)$	$E(X)$	$\text{Var}(X)$	$M_X(t)$
Binomial	n, p	$\binom{n}{i}p^i(1-p)^{n-i}$	np	$np(1-p)$	$(pe^t + 1 - p)^n$
Poisson	λ	$e^{-\lambda}\frac{\lambda^i}{i!}$	λ	λ	$e^{\lambda(e^t-1)}$
Multinomial	$n, p_1, p_2, \ldots, p_k$	$\binom{n}{i_1, \ldots, i_k}p_1^{i_1}\cdots p_k^{i_k}$			

Continuous Random Variables

See table on Page 3.

- Hazard rate function - $\lambda(t) = \frac{f(t)}{1-F(t)}$.

Joint Distributions

- Sums of independent random variables:
  - continuous: $f_{X+Y}(x) = \int_{-\infty}^{\infty} f_X(y)f_Y(x-y)dy$
  - discrete: $p_{X+Y}(x) = \sum_y p_X(y)p_Y(x-y)$

- Transformation of variables from x_1, y_1 to x_2, y_2:

  1. Jacobian $J = \begin{vmatrix} \frac{\partial x_2}{\partial x_1} & \frac{\partial x_2}{\partial y_1} \\ \frac{\partial y_2}{\partial x_1} & \frac{\partial y_2}{\partial y_1} \end{vmatrix} = \frac{\partial x_2}{\partial x_1}\frac{\partial y_2}{\partial y_1} - \frac{\partial y_2}{\partial x_1}\frac{\partial x_2}{\partial y_1}$.
  2. $f_{X_2,Y_2}(x_2, y_2) = f_{X_1,Y_1}(x_1, y_1)|J|^{-1}$ or $f_{X_1,Y_1}(x_1, y_1) = f_{X_2,Y_2}(x_2, y_2)|J|$.

Inequalities

- Markov's inequality $P(X \geqslant a) \leqslant \frac{E(X)}{a}$.
- Chebyshev's inequality $P(|X - \mu| \geqslant a) \leqslant \frac{\sigma^2}{a^2}$.
- One-sided Chebyshev inequality $P(X \geqslant a) \leqslant \frac{\sigma^2}{\sigma^2 + a^2}$.
- Chernoff bounds:
  - $P(X \geqslant a) \leqslant e^{-ta}M(t)$ for $t > 0$.
  - $P(X \leqslant a) \leqslant e^{-ta}M(t)$ for $t < 0$.

Distribution	Parameters	Probability density function	cumulative distribution function $F(x)$	$E(X)$	$\mathrm{Var}(X)$	Moment generating function
Uniform	a, b	$\begin{cases} \frac{1}{b-a} & \text{if } a < b < x \\ 0 & \text{otherwise} \end{cases}$	$\begin{cases} 0 & x < a \\ \frac{x-a}{b-a} & a < b < x \\ 1 & x > b \end{cases}$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$	$\frac{e^{tb}-e^{ta}}{t(b-a)}$
Normal	μ, σ^2	$\frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$	$\Phi(x)$ (see table)	μ	σ^2	$e^{\mu t + \frac{\sigma^2 t^2}{2}}$
Exponential	λ	$\lambda e^{-\lambda x}$	$1 - e^{-\lambda x}$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$	$\frac{\lambda}{\lambda - t}$ (for $t < \lambda$).