

MATH/STAT 3360, Probability  
FALL 2013  
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Sample Midterm Examination

This Sample Midterm has more questions than the actual midterm, in order to cover a wider range of questions.

1. How many distinct ways can the letters of the word “ EXAMINATION” be arranged?
2. What is the probability that the sum of 3 fair 6-sided dice is 6?
3. Calculate the probability density function of the square of an exponential random variable with parameter  $\lambda$ .
4. What is the probability that a five-card poker hand is a flush (all cards the same suit)?
5. A fair coin is tossed 7 times. What is the probability that the sequence HHTT occurs somewhere in the 7 tosses?
6. Three coins are tossed: are the following events independent?
  - (i) The first two tosses are both heads.
  - (ii) The total number of heads is odd.
7. Suppose the number of cars that want to park in a particular street each day is a Poisson random variable with parameter 4. There are 3 parking spaces on the street.
  - (a) What is the probability that the number of cars parking on that street is exactly 2?
  - (b) What is the probability that all the parking spots are taken?
  - (c) What is the expected number of free parking spaces?
8. A patient is given a routine test for a rare disease. The disease affects 3 people in 1000. The test is 98% accurate, so there is a 2% chance of giving the wrong result. The test result is positive (i.e. indicates the patient has the disease). What is the probability that the patient actually has the disease?
9. A company is conducting a survey. They want to determine the proportion of people who would buy their new product. The true proportion is 30%.  
[You may use any reasonable approximations for the distribution of the proportion of people surveyed who say they would buy it.]

- (a) If they survey 300 people, what proportion has a 95% probability of being larger than their estimate? That is, find  $x$  so that there is a 95% probability that their estimate is less than  $x$ .
- (b) how many people do they need to survey so that the probability that their estimate is within 2% of the true value (i.e. between 28% and 32%) is at least 95%?
10. A company makes light bulbs. The company has two machines for making them. light bulbs made by one machine have lifetime (in years) exponentially distributed with parameter 2, and light bulbs made by the other machine have lifetime exponentially distributed with parameter 3. 30% of its products are made by the first machine.
- (a) What is the probability that a randomly chosen light bulb lasts for at least 1 year?
- (b) Given that it lasts for 1 year, what is the probability that it was produced by machine 1?
- (c) Given that a light bulb has lasted for 1 year, what is the probability that it lasts for a second year?
11. An insurance company sells 12,000 policies, each of which has probability  $\frac{1}{8,000}$  of resulting in a claim. What is the probability that it receives more than 2 claims?
12. A company produces 50,000,000 scratchcards, which it will sell for \$1 each. The scratchcards offer the following cash prizes:

Prize	Number of cards with this prize
0	48,000,000
\$10	1,850,000
\$100	140,000
\$1,000	9,750
\$10,000	235
\$100,000	14
\$1,000,000	1

What is the expected value and variance for the prize of a scratch card?