

MATH/STAT 3360, Probability  
FALL 2013  
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Midterm Examination  
Thursday 24th October: 1:05–2:25 PM

Each part question (a, b, c, etc.) is worth 1 mark.

1. What is the probability that a five-card poker hand is a two-pair (two cards of one rank, two of another, and one of a third rank)?
2. A fair coin is tossed 8 times. What is the probability that the sequence THTT occurs somewhere in the 8 tosses?
3. Four fair coins are tossed. Are the following events independent:
  - (i) The first toss is a head
  - (ii) There are exactly two heads.
4. A patient is given a routine test for a rare disease. The disease affects 1 person in 5000. The test is 99% accurate, so there is only a 1% chance of giving the wrong result. The test result is positive (i.e. indicates the patient has the disease).

What is the probability that the patient actually has the disease?
5. A company is conducting a survey. They want to determine the proportion of people who would buy their new product.
  - (a) If the true proportion is 40%, and they survey 300 people, what is the probability that their estimate is within 1% of the true value (i.e. between 39% and 41%)?

[You may use any reasonable approximation for the distribution of the number of people who say they would buy this product. You may also assume that the total number of people who could be surveyed is large enough that different peoples responses are independent.]
  - (b) Find the smallest interval such that the probability that the true proportion is in this interval is at least 95%.
  - (c) How many people do they need to survey so that the probability that their estimate is within 1% of the true value is at least 95%?
  - (d) [bonus] How many people do they need to survey to be sure that whatever the true proportion is, their estimate is within 1% of it with probability at least 0.95.
6. A company makes computers. The lifetime of the computers it makes (in years) is exponentially distributed with parameter 0.3. You upgrade your computer every 3 years. What is the probability that a computer from this company breaks down before you are ready to upgrade it?

7. The number of customers a company has on a given day is a Poisson random variable with parameter 3.6. What is the probability that the company receives exactly 5 customers on a given day?
8. A company produces 80,000,000 scratchcards, which it will sell for \$1 each. The scratchcards offer the following cash prizes:

Prize	Number of cards with this prize
0	78,000,000
\$10	1,840,000
\$100	145,000
\$1,000	13,600
\$10,000	1,385
\$1,000,000	15

What is the expected value and variance for the prize of a scratch card?