MATH/STAT 3460, Intermediate Statistical Theory
Winter 2014
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Midterm Examination
Monday 3rd March 14:35–15:25

1. The weight (in kg) of a certain species of rabbit is believed to follow a Normal distribution with mean 5 and variance $7e^{-2\sqrt{a}}$. Eight of these rabbits are collected, and their weights are measured as 4.9, 6.8, 3.6, 8.1, 2.3, 3.1, 6.4, and 4.2. What is the maximum likelihood estimate for a?

2. In a trial for a new drug, the probability of a response to dose d is assumed to be $1-\frac{1}{1+e^{\alpha+\beta d}}$ for some α and β. The data from a study of the drug are given in the following table:

dose	0	1	2
number	26	23	21
number of responses	3	12	21

   (a) Show that $\alpha=-2.40315$ and $\beta=2.78828$ is the maximum likelihood estimate for α and β.

   (b)[**Bonus**] Use a normal approximation to calculate a 10% likelihood region for (α, β).

3. The remaining lifetime (in years) of a patient undergoing a certain kind of treatment is exponentially distributed with parameter λ. In a study which follows 10 patients for a period of 3 years, seven of the patients have lifetimes: 0.3, 0.8, 0.9, 1.4, 1.8, 2.5, and 2.9, while the remaining three patients survive to the end of the three-year period. What is the maximum likelihood estimate for λ?

4. We observe two samples from a Poisson distribution with parameter λ. If the true value of λ is 0.7, what is the probability that this value lies within a 10% likelihood interval? [Calculate the exact coverage probability.]

5. The probability of a particular genetic condition is $p=\theta^2$. Let N be a sample from a binomial distribution with parameters 100 and θ. The MLE for p is $\left(\frac{N}{100}\right)^2$. What is the bias of this estimate?