ACSC/STAT 3703, Actuarial Models I (Further Probability with Applications to Actuarial Science)

WINTER 2015

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Midterm Examination

Monday 2nd March

9:35-10:25

1. Let X have density function given by

$$f(x) = \frac{(x-2)e^{\frac{x}{\theta}}}{\theta^2 - 2\theta}$$

for $0 < x$.

(a) Show that the distribution of X is from the linear exponential family, and calculate the functions $p(x)$, $q(\theta)$, and $r(\theta)$.

(b) Calculate the variance of X as a function of θ.

2. You observe the following sample of insurance losses:

1.6 2.6 3.1 3.9 4.8

Using a Kernel density model with a uniform kernel with bandwidth 2, estimate the TVaR at the 95 % level for the claim distribution.

3. The value of car owned by a driver follows a gamma distribution with $\alpha = 2$ and $\theta = 3000$. Given that a driver drives a car of value Θ, the distribution of the size of claim made on the insurance policy follows an inverse Weibull distribution with parameters $\tau = 2$ and $\theta = \Theta$. What is the probability that a randomly chosen claim exceeds \$30,000. [Hint: $\int_0^\infty xe^{-\frac{x^2+2ax}{b^2}}dx = \frac{b^2}{2} - ab\sqrt{\pi}\left(1 - \Phi\left(\frac{a\sqrt{2}}{b}\right)\right)e^{\frac{a^2}{b^2}}$.]

4. Claim frequency from 500 policies follows a compound Poisson-Negative binomial distribution with $\lambda = 10$, $r = 4$ and $\beta = 1.8$. The following year, the number of policies decreases to 300. Calculate the probability that there are exactly 2 claims the following year.