ACSC/STAT 3703, Actuarial Models I (Further Probability with Applications to Actuarial Science) Winter 2015

Toby Kenney

Homework Sheet 2

Due: Friday 23rd January: 12:30 PM

Basic Questions

1. Calculate the probability density function of a random variable that is 7 times a beta random variable with $\alpha = 3$ and $\beta = 2$. The density function of this beta random variable is

$$f_X(x) = \begin{cases} x^2(1-x) & \text{if } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

2. Calculate the distribution of X^8 when X follows a gamma distribution with $\alpha = 3$ and $\theta = 13$.

3. X is a random variable with moment generating function $M_X(t) = \frac{1}{(3-t)\left(1-\frac{t}{6}\right)}$. What is the variance of the random variable e^X?

4. X is a mixture of 3 distributions:

   - With probability 0.2, X follows a gamma distribution with $\alpha = 2$ and $\theta = 2000$.
   - With probability 0.35, X follows a gamma distribution with $\alpha = 3$ and $\theta = 4000$.
   - With probability 0.45, X follows a Weibull distribution with $\theta = 2000$ and $\tau = 4$.

   The moments of these distributions are given in the following table:

	Distribution 1	Distribution 2	Distribution 3
μ	4000	12000	1812.805
μ_2	80000000	48000000	258645.631975
μ_3	3.2×10^{10}	2.56×10^{11}	11474411.56287975
μ_4	3.84×10^{14}	1.152×10^{16}	183821938794.038572798
μ'_2	2.4×10^7	1.92×10^8	3544907.60000
μ'_3	2.56×10^{11}	5.44×10^{12}	13309852126.945560125
μ'_4	2.944×10^{15}	1.6896×10^{17}	59198070889950.1844896020

   (a) What is the coefficient of variation of X?

   (b) [bonus] What is the kurtosis of X?

5. For a particular claim, the insurance company has observed the following claim sizes:

   12.3, 16.8, 24.6, 25.2, 25.4, 25.8, 30.2, and 35.3.

   Using a kernel smoothing model with a Gaussian kernel with variance 0.5, calculate the probability that the next claim size is between 22 and 26.

Standard Questions

6. An insurance company finds that the loss experienced by an individual follows an inverse exponential distribution with θ depending on the individual. It models this θ as following a gamma distribution with $\alpha = 3$ and $\theta = 2000$. What is the distribution of the loss of a random individual.

7. A life insurance company models the mortality of an individual as following a Gompertz law with hazard rate given by $\lambda = 0.00001ae^{0.1t}$, where a is the frailty of the individual. It models a as following a gamma distribution with $\alpha = 0.4$ and $\theta = 2$. Calculate the probability that a randomly chosen individual lives to age 100.

8. An insurance company wants to model a random variable X. It believes that for large values, it should use a Pareto distribution with $\alpha = 4$ and $\theta = 300$ to model the distribution of values above 5000. For values below 5000, it plans to use an inverse gamma distribution with $\alpha = 3$ and $\theta = 800$. If 5% of values are above 5000, what is the probability under this model that the value of X is between 3000 and 10000?