

ACSC/STAT 3703, Actuarial Models I (Further
Probability with Applications to Actuarial Science)
Winter 2015
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Homework Sheet 3
Due: Monday 9th February: 12:30 PM

Basic Questions

1. Construct a distribution which is a special case of all distributions in the transformed beta family. How many parameters does it have?
2. If X follows a log-logistic distribution with $\gamma = 3$ and $\theta = 400$, what is the distribution of $\frac{1}{X}$?
3. What is the limit of a generalised Pareto distribution as $\alpha \rightarrow \infty$ and $\theta \rightarrow \infty$ with $\frac{\theta}{\alpha} \rightarrow \xi$?
4. Calculate the skewness of a linear exponential distribution with pdf

$$f_X(x) = \frac{p(x)e^{r(\theta)x}}{q(\theta)}$$

Standard Questions

5. What is the limiting distribution of an inverse transformed gamma distribution

$$f(x) = \frac{\tau \left(\frac{\theta}{x}\right)^{\alpha\tau} e^{-\left(\frac{\theta}{x}\right)}}{x\Gamma(\alpha)}$$

as $\alpha \rightarrow \infty$, $\tau \rightarrow 0$ and $\theta \rightarrow 0$, with $\frac{\sqrt{\theta\tau}}{\tau} \rightarrow \sigma$ and $\frac{\theta^\tau \alpha - 1}{\tau} \rightarrow \mu$?

6. An insurance company is modelling claim size by a distribution from a linear exponential family with one parameter θ , with density function

$$f_X(x) = \frac{p(x)e^{r(\theta)x}}{q(\theta)}$$

on its support, $[0, \infty)$. The company wants the mean of the distribution to be θ , and the variance to be $\frac{\theta^2}{3}$ for all values of θ . What is the pdf of the distribution? [Hint: $p(x)$ will be a quadratic function of x . Try evaluating the integral for $p(x) = x^2$, $p(x) = x$ and $p(x) = 1$ to find the correct function for $p(x)$.]