

ACSC/STAT 3703, Actuarial Models I (Further
Probability with Applications to Actuarial Science)
Winter 2015
Toby Kenney
Homework Sheet 4
Due: Monday 23rd February: 12:30 PM

Basic Questions

1. Let X follow a negative binomial distribution with $r = 4$ and $\beta = 1.2$. What is the probability that $X = 8$?
2. The number of claims on each insurance policy over a given time period is observed as follows:

Number of claims	Number of policies
0	736
1	382
2	74
3	7
4	2
5 or more	0

Which distribution(s) from the $(a, b, 0)$ -class and $(a, b, 1)$ -class appear most appropriate for modelling this data?

3. X follows an extended modified negative binomial distribution with $r = -0.8$ and $\beta = 2$, and $p_0 = 0.4$. What is $P(X = 7)$?
4. Let X follow a compound Poisson-Negative binomial distribution with parameters $\lambda = 3.3$, $r = 4.8$ and $\beta = 2.3$. Calculate the conditional probability that $X = 7$ given that $X \leq 10$.
5. Let X follow a mixed negative binomial distribution with $\beta = 1.5$ and r following a gamma distribution with $\alpha = 2$ and $\theta = 4$. What is the probability that $X = 2$?

Standard Questions

6. An insurance company estimates that the number of claims made by an individual in a year follows a Poisson distribution with parameter λ , where λ varies between individuals, following a gamma distribution with $\alpha = 3$ and $\theta = 0.05$.
 - (a) What is the probability that a randomly chosen individual makes 3 claims in a given year?

- (b) If an individual has made 3 claims in a given year, what is the probability that that individual makes 3 claims in the next year?
7. An insurance company models the number of claims X on a given policy using a distribution from the $(a, b, 1)$ -class. The company wants its distribution to match the observed mean $\mathbb{E}(X) = 0.475$ and probability of zero $P(X = 0) = 0.738$, and also wants $P(X > 3) = 0.01$. From this, they calculate $P(X = 1) = 0.1120652294$. Under this model, what is the probability that an individual makes 4 claims in a year? [Hint: for a general member of the $(a, b, 1)$ - class, we have $\mathbb{E}(X) = \frac{p_1 + (a+b)(1-p_0)}{1-a}$ and $p_1^T = \frac{a+b}{(1-a)^{-1-\frac{b}{a}} - 1}$.]
8. An insurance company insures 200 houses. The number of claims resulting from these policies follows a compound Poisson-Binomial distribution with $\lambda = 12$, $n = 8$ and $p = 0.001$. The company's risk management division wants to ensure that the probability of receiving 2 or more claims should be at most 0.001. How many houses can the company insure while satisfying this condition?
- (i) 52
(ii) 88
(iii) 147
(iv) 260

Bonus Question

9. Using the general recursion formula, show that the expected value of a distribution from the $(a, b, 0)$ -class is given by $\frac{a+b}{1-a}$.